A New Back Casting Method for Valuing Urban Land: Comparison to Land Residuals*

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Abstract

We develop a new method for urban land valuation based on theory which implies that land and structure trade as a bundle until the structure has no economic value. This back casting method first estimates property value, construction costs and residual land value in the year of new construction. Thereafter, the ratio of land to property value changes primarily with structure depreciation; changes in property value are shared by land and structure components. In contrast, land residual methods (land value equals property value minus the depreciated cost to rebuild in the sales year) predicts that the ratio is volatile because it is leveraged by relatively stable replacement costs.

We fit both models to Maricopa County assessor data on houses up to 28 years old during a bust and recovery period (2007–2018) and we evaluate the models for relevance to property tax assessment. Our inability to distinguish from a counterfactual points towards future research focused on sample selection.

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Highlights

- Our model of urban land value modifies Alonso-Muth-Mills (AMM) theory by adding the assumption that land and structure trade as a bundled good in the years immediately after construction.
- We develop a new back casting algorithm for measuring the ratio of land to total value at construction, updated with structural depreciation.
- Back casting is compared to a land residual method which says that land and structure values can evolve independently immediately after construction.
- We evaluate these models with single family sales data for Maricopa county, Arizona, 20007-2018 using criteria relevant to assessment practice.
- For tax assessment purposes, data and theory support a hybrid model which allows some independence between land and structure values but not as much as under the land residual model.
- Models of structure value are not supported by the data, and we cannot distinguish land value models from a counterfactual, suggesting further research on the selection of locations for new construction.

1. Introduction

Appraisers and tax assessors use a hypothetical "as if vacant" definition of land value which ignores the high cost of tearing down existing structures[1]: These costs exceed direct demolition costs since the rental value of the existing structure must be sacrificed to exchange it for a new structure that is at highest and best use (HBU). Appraisers recognize this by allowing the property to be valued as improved. The HBU for most properties is as improved: irreversibility, the high cost of giving up the value of existing structures, implies that teardowns and redevelopment are typically infrequent and highly localized.

In this paper we define urban land value as the value at the time a structure was built adjusted for changes in property value over time and adjusted for changes due to possible demolition, renovation or redevelopment of the depreciated structure.[2] Importantly, our definition allows structure and land values to interact in complex ways as the property progresses from new construction to demolition and redevelopment.

In this study, we apply our definition to newly constructed properties where irreversibility due to slow depreciation of structures is most relevant.[3] Slow reversibility implies that the new structure and land will trade as a bundled good, implying that both will be influenced proportionally in the same

1 The Appraisal Institute (2008, p281) says "Land is generally valued as though vacant. … When land is not vacant, however, its contribution to the value of the property as improved depends on how it can be put to use." Here we discuss valuation methods contained in Chapter 16, "Land and Site Valuation."

2 A possible change in the structure means legally, physically and financially feasible now or at some future time.

direction by changes in the valuation of the bundle, even over the first 10 or 20 years of structure life. A gradual change in the land value ratio (i.e., land value divided by property value at any time) due to structure depreciation is the most important empirical prediction of our new definition.

Our concept of land value is based on long standing theory based on models by Alonso (1964), Muth (1969) and Mills (1972) (hereafter, "AMM" theory) which says that property values derive from the present value of net rents generated from the structure and its location. In AMM theory, land values are dependent on a structure which is built so as to maximize the present value of the location, i.e., HBU structure. Land value at the time of new construction is a residual equal to the HBU property value less construction costs.

Scholars generally agree that AMM theory holds at the time of new construction and there is also agreement that it holds just before redevelopment when the property trades primarily for land value, and that value is determined by property value with a HBU structure. Our new idea is that AMM theory does not hold in the years immediately after HBU construction because land and structure trade as a bundled good: i.e., relatively gradual reversibility governs the land value ratio.

A broad outline of land valuation based on back casting

We propose to measure urban land value with a back casting algorithm which provides new empirical content to one of the three existing methods used by professional appraisers for land valuation:[4] According to the land allocation method, land value is defined as some percentage of property value. The problem with professional practice is that the percentage allocated to land often lacks theory and empirical support when applied to most urban properties. For example, assessors and appraisers might use a constant percentage – or one that does not vary much across time or space. Our data for Maricopa County show that the assessor used a ratio of exactly 0.2 for over 98% of single family residential (SFR) properties valued in 2017.

To revise the allocation method, we start with a standard hedonic valuation equation which is basically an automated valuation model (AVM) similar to those used for mass property valuation. An

[3] Our focus on new construction, and our development of the back casting method for land valuation, differentiates this paper from Clapp et al. (2021) where option value theory and empirical results include all age categories.

[4] See Table 16.1 in the Appraisal Institute (2008) for a concise summary of these three valuation methods. The other two are: 1) the value of the property minus cost to replace the depreciated structure (the land residual model, also known as the "market extraction method;" (2) using sales of vacant land or properties purchased for teardown and redevelopment ("sales comparison method").

AVM is the computer assisted part of computer assisted mass assessment (CAMA) methods on which we focus.

The AVM model produces a local house price index (HPI) which we use to back cast AVM values from the year of sale to the year of construction. Then we use information from a construction cost manual to estimate the cost of each structure at the time it was built. If the structure were built to HBU at the time of construction, AMM theory says that construction cost equals structure value. Land value is the AVM property value minus HBU structure value, both at the date of construction.

The next step is to use an estimate of structure depreciation to update the land value ratio from the time of construction to the date of any sale of the property. Multiplying this ratio by AVM value as of the sales date provides an estimate of land value. Accurate estimates of depreciation are important to the real economic value of structure, an estimate of reduction in property value if the structure should be destroyed by natural causes.

An important reason for modelling land value ratios is to estimate land value for all properties (e.g., for tax assessment), even those not sold or not available for construction cost estimates. It is possible to extrapolate land value ratio an corresponding land value estimates to a broad class of properties not used to estimate the model. For example, we propose to use a model of land value ratios fitted to construction since 1999 to extrapolate to all properties built and sold since 1989.

The land value ratio needs to be modelled in order to deal with the possibility that individual structures have not been developed to HBU, i.e., mistakes are made by developers ,or that the market is not in equilibrium due to mispricing by buyers an sellers, possibly resulting in negative ratios for new properties. For example, during the global financial crisis years (2007–2011) we found that the volume of new construction declined by over 75% from its earlier levels. Still, some houses were built and sold, most at distressed price levels. In these cases, naïve application of the back casting method produced very low or negative land value ratios in the year of sale. We propose methods for modelling land value ratios to deal with market disequilibrium and/or mistakes by developers and home owners.

Land residual theory and methods

The widely-applied land residual method estimates structure value as equal to the cost to build a new structure at the time of the sale less depreciation. Like back casting, land residual theory is based on AMM theory: at the time of new construction land must be purchased at a value determined by HBU property value minus construction costs. At this point property value is the sum of land plus structure values just as in the back casting method.

Land residual theory holds that structure value changes over time as construction costs change, less depreciation which is usually conceptualized as physical wear or functional obsolescence which reduces structure value.[5] Since property value is driven by changes in supply and demand, this assumption allows land and structure to evolve largely independently, even in the years immediately after construction. While the theory acknowledges that depreciation can become increasingly difficult to estimate as the structure ages, its most recent iteration claims that it holds at least during the first 15 years of structure life. This study contrasts back casting with land residual values for new structures.[6]

Criteria for evaluating back casting vs. land residual methods

We have rigorous criteria for evaluating the performance of back casting and land residual methods:

1. The division of property value into land and structure must produce values that are easily explained and justified to tax payers when used to tax land at a different rate than structures. Any shifting of the tax burden over time should be based on facts that can be explained and justified.[7]
2. Valuation methods must be suitable for CAMA application. Algorithms should be applicable to all sales with minimal judgement about eliminating outliers, etc.
3. Models of land value ratios must be robust to extrapolating values outside the sample used to estimate them.[8]
4. The algorithms should not increase Coefficients of Dispersion (CODs), a widely applied measure of assessment accuracy calculated from the ratio of assessed value to sales prices.[9] If structure value can

[5] A thorough exposition of land residual theory is in chapters 5 and 27, Geltner *et al.* (2006).

[6] This differs from our companion paper, Clapp *et al.* (2021) which compares valuation based on options theory to land residuals using market-wide averages, not land valuations at the individual property level. The back casting algorithm is new to this paper.

[7] Tax assessors sometimes have to tell owners that property values in their neighborhood went up, so tax bills shift towards them. They explain this by pointing to known sales prices. Similarly transparent justifications would be needed for split taxation.

[8] Davis *et al.* (2019) use land residual and kriging methods to extrapolate land value ratios from their estimation sample to all single family.

[9] COD's are a nonparametric version of standard deviations and coefficients of variation in the ratio of assessed values to sales prices. COD's evaluate assessment accuracy around the median ratio, accounting for accuracy in the tails of the distribution.

be successfully modeled separately from land value, the sum of structure and land should reduce CODs when compared to a CAMA model that does not separate land and structure.

Criteria 1–3 are relevant to land valuation for the purposes of taxing land and structure at different rates. I.e., reliable land value ratios that can be explained to property owners during volatile market periods could be used as a basis for equitable property taxation. These ratios can be multiplied by existing property valuations quickly and transparently, without negatively impacting CODs.

We introduce criterion 4 to test for the real economic value of structures which we define as the reduction in property value that would be observed if the structure were destroyed by fire or other natural event. Since destruction is not observed in our data (and atypical of many datasets) we use CODs based on separate structure and land models compared to CODs based on AVMs. Criterion 4 is much more demanding of modeling accuracy because it requires replacing existing assessed values with those based on separate land and structure values. Without a model of structure value, land and structure would add up to AVM property value at the sales date by definition: land value equals the AVM value times the land value ratio and structure value is what is left over.

Estimating land value for new construction: the first part of a "three buckets" method

This paper focuses on new construction defined as any structure where there is no significant value in the option to redevelop, implemented here with data on properties less than 28 years since construction. This includes a broad class of properties in most markets since it is unusual for values in a local market to change so much that 30- or 40-year-old properties are being torn down or substantially redeveloped. We will point to data suggesting that after structure age approaches 30 or 40 years then net depreciation approaches zero: i.e., maintenance and renovation begin to dominate depreciation and obsolescence becomes an issue.

In a broader context, we view land valuation as being implemented with a "three buckets" method. This paper deals with the first bucket, which is the valuation of land and structure for relatively new structures. The third bucket is properties approaching the end of their economic lives where back casting will not apply because they are too far from new construction. In these properties depreciation should be defined to include obsolescence and, for structures approaching the end of life, option value. The second bucket is properties that have some obsolescence but they are too far from construction to use back casting. We leave exploration of the last two buckets to future research.

We develop the back casting method using Maricopa County assessor data provided to the Lincoln Institute for Land Policy for research purposes. The Maricopa county assessor provided extensive data on sales of vacant land (2000–2018) and sales of properties with structures (2007–2018). Importantly, the assessor has divided the county into market areas and each market area is divided into neighborhoods in recognition of the fact that values can differ substantially across space as well as time.

2. Literature Review

This section summarizes the most relevant literature related to land valuation related to bundled-good and land residual methods.[10] A more comprehensive literature review is in Clapp, Cohen and Lindenthal (2021).

The land residual method – where land value equals the difference between property value and the cost of replacing the structure, both estimated at the time of sale – dominates scholarly research and applied methods for separating land and structure values. The assumption that structure is valued at cost to reproduce (cost to build new, less depreciation) combined with relative ease in calculating construction costs leads to widespread use of the model. Davis and Palumbo (2008) develop a land price index, and estimate it across MSAs and time from 1975. Davis *et al*. (2019) develop land and structure value estimates with broad U.S. geographical coverage at the zip code and census tract levels, annually 2012-2018.

Estimating depreciation and obsolescence for older properties poses a challenge to the land residual method. Davis *et al*. (2019) deal with this by confining their estimates to properties with structures built within 15 years of the sales date. They obtain professional cost appraisals which are typically done for the purpose of mortgage underwriting. By limiting land and structure value estimates to newly constructed properties, they eliminate the need to estimate depreciation and obsolescence for older properties, an approach that we employ also.

Some studies have not been so careful about dealing with depreciation. Knoll *et al*. (2017) use the residual land value method to study historical land values for 14 different countries, going back to the late 1800s in some cases. Their sources and methods differ by country, but in most they apply the land residual model to all structures regardless of age. For the U.S., they find that land values account for approximately 80% of the house price appreciation since World War II. They find that the ratio of land to total value has increased substantially since 1950 in many countries.

[10] The bundled good literature can be taken as synonymous with hedonic valuation models which are widely used for property appraisal and tax assessment.

We follow Davis *et al*. (2019) by focusing on new construction, an approach that is usefully compared to literature focusing on teardown properties which are close to the point of new construction: i.e., land value is approaching 100% of property value. Dye and McMillen (2007) and Helms (2003) use Chicago data to evaluate the impact of location and structure characteristics on the potential for teardown. Munneke and Womack (2017) find teardowns are concentrated in space and time, as well as further evidence that structure value is approaching zero for these properties. Clapp and Salavei (2010) examine the intermediate case of properties that have some option value but are not near the point of teardown, as is true of a significant share of urban property. This is consistent with the depreciation literature discussed below: properties over 40 or 50 years old typically contain some option value, or the option has been exercised through substantial renovation or teardown and replacement.

One conclusion from the option value literature is that the economic value of structures approaches zero as the structure gets closer to teardown. Munneke and Womack (2017) develop a clever method for determining the percentage of structure value remaining as a function of structure age, characteristics and location. Importantly for our study, the economic value of structure will differ dramatically from the cost to replace in these cases unless obsolescence is taken into account: the land residual literature has not produced a good way for correcting construction cost estimates for option value. The present study focuses on new construction as a way to avoid dealing with the difficult problem of estimating depreciation and obsolescence.

Bostic *et al*. (2007) initiated the land leverage literature, developing empirical evidence of high land volatility, evidence that was amplified by Knoll *et al*. (2017). An important part of the land leverage hypothesis is that the riskiness of real estate is an increasing function of the ratio of land value to total property value, a proposition effectively argued by Bourassa *et al*. (2011).

A problem with the land leverage hypothesis is that causation is assumed to run from land value to total value, a logical consequence of the assumption that structure is valued by slowly changing replacement cost while real estate values are well-known for high amplitude fluctuations. Bostic *et al*. (2007) discuss applications to measuring risk associated with public and private decisions. But theory to be developed below says that property value is derived from net rent on structures, and that land value is a residual after subtracting the cost of new construction to the HBU level. We think this is contradicted by the land leverage assumption that most shocks to property value are to land value, not to economic structure value.

The age-value profile and structure depreciation

The back casting method requires a parameter for annual structure depreciation because it uses cost and property value at the time of construction to estimate the land value ratio, then updates the ratio to the date of sale with structure depreciation. Similarly, the land residual model applies depreciation to the cost to build at the date of sale. We rely on previous literature for a range of depreciation estimates and for the choice of a functional form for depreciation.

Hedonic valuation models typically include a polynomial in structure age, with results typically showing an age-value profile that declines at a decreasing rate, at least for the first 20 to 40 years of structure life that are of primary concern here. This is sometimes referred to as an estimate of the rate of depreciation and obsolescence, but that view ignores the fact that land does not depreciate. [11]

Back casting requires estimates of annual structure depreciation net of typical maintenance and improvements. Goodman-Thibodeau (1995) point out that common home improvements (new kitchen, bath, HVAC or roof) are typically not observed by the econometrician, leading to heteroscedasticity in the estimated age-value profile. Since these typically enhance property value, hedonic age coefficients can be interpreted as reflecting net change in the profile.[12]

Goodman-Thibodeau (1995) use cubic and quartic function of age to evaluate the value-age profile in a standard hedonic model. They are talking about net depreciation as reflected in the value-age profile when they say that value declines by around 3.5% per year for new structures, less than half that for 8-year-old structures, near zero for 15–20-year-old and slight appreciation for 20–40-year-old structures (p 40). These findings are roughly consistent with non-parametric apartment building results in Bokhari and Geltner (2019). In particular, they find substantial value declines in the first few years of a structure life, analogous to the decline in the value of an automobile once it is no longer new.

[11] Depreciation is best conceptualized as a change in the quantity of structure, not its value, because, in AMM theory, land value is derived from net rent per unit structure which should not change between small and large structures at identical locations except for spreading of overhead over structure size.

[12] In private communication, Professor John Harding uses American Housing Survey (AHS) data (over 60,000 observations) to regress maintenance and improvement expenditures as a percent of property value on interior square footage, number of rooms and a polynomial in building age. The signs of age coefficients are a mirror image of those in cubic and quartic regressions in Goodman and Thibodeau (1995) suggesting that owners typically offset some part of gross depreciation. It follows that age coefficients in a typical hedonic regression will measure net depreciation. AHS maintenance data include major renovations to kitchen, bath, roof, HVAC and the like.

We double these value-age numbers based on our data showing that structure is roughly half of new property value in our high valued market. This gives 2% to 4% per year structure depreciation averaged over 20 to 30 years; these rates are net of maintenance and improvements and they apply specifically to the structure. We chose an exponential functional form based on exponential approximations to non-parametric value-age profiles examined by Bokhari and Geltner (2019). This has the advantage of simplicity in a study which is focused on land value, a quantity that does not depreciate.

AMM theory

Alonso (1964), Muth (1969) and Mills (1972) develop theory for a monocentric city on a featureless plain under competitive market assumptions.[13] Alonso (1964) starts with households that need a place to live with access to some desirable point such as the central business district (CBD) where they work. He starts with a utility function for housing and a composite of other goods and a budget constraint that includes the cost of commuting to the CBD. His great insight was that utility drops out of a solution where housing costs change inversely to commuting costs, an example of compensating differentials.

Muth (1969) builds on this insight in chapter 2, considering household locations as a function of income and taste and developing an early version of rent functions as a result of equilibrium envelope of competing utility functions. Of most relevance to our study, Muth says "in my framework the demand for residential land is treated as derived from the demand for housing rather than as a demand at the consumer level, as in other studies (p. 18)." In Chapter 5 Muth allows for the demand for land and structures to be different functions of income, providing foundations for the hybrid model we develop below.

Both Muth (1969) and Mills (1972) excel at taking their models to the data with numerous insights about urban form, notably showing how land values, land uses (e.g., office, industrial and residential), and population and structural density vary with distance to the CBD. We are particularly interested in analysis of equilibrium rents and quantities between consumers and producers at a given location (i.e., a given monocentric anulus): Richardson (1977, chapter 3) has an excellent mathematical treatment part of which, like our model, shifts away from urban form and focuses instead on land rents.[14]

[13] A critical review of the many assumptions required by AMM theory, and of advances from relaxing some assumptions, is in Richardson (1977). For example, he points out that housing must be produced by firms and employment distributed throughout the city, contradicting commuting exclusively to the CBD.

[14] For a nontechnical (graphical) treatment clearly showing land rents as a residual holding constant for location, see Heilbrun (1987), chapter 6. He carefully lays out many of the model assumptions on page 108.

3. A simple theoretical model and numerical example

Land residual theory is based on AMM assumptions that renovation and replacement of structures keep them near their HBU value, holding constant for location (i.e., distance to the CBD). At the time of construction, AMM theory holds that property value is driven by the present value of net rental income and that structure size is determined to maximize property value. Land value is a residual: property value minus the cost to build to HBU. In the years immediately after construction, land residual theory holds that structure value is depreciated replacement cost: i.e., except for depreciation the AMM continue to hold, at least until the structure is 10 or 15 years old.[15]

Our model departs from land residual theory by assuming slow reversibility due to depreciation – an assumption referred to as irreversibility in the literature – which implies that, after new construction, land and structures trade as a bundled good: AMM assumptions hold at the time of construction but not when expected net rents change after that date.[16] With bundled good assumptions (irreversibility) structure value as a ratio to property value changes slowly due to depreciation after construction. We are interested in compensating differentials between decline in structure value with depreciation and land and structure value, holding constant for location. We focus on the years immediately after construction when there is typically little options value. This allows relatively simple analysis of structure value over time until the structure has reached a point where obsolescence greatly reduces its value.[17]

As in Muth (1969) and Mills (1972) units of housing are produced with land, L and structure, S which are measured in the same units (e.g., square feet), except that we simplify to a linear production function. Each unit of housing (stock of housing, H) delivers one unit of services per time period:[18]

15 See Davis *et al.* (2019) for an explanation of the 10 or 15 year assumption.

16 Muth (1969) addressed durable structures. "the relative worth of comparative static analysis versus historical analysis" where durable structures are given by history. Because of his focus (along with most analysts using the AMM model) on urban form he concludes that "long-run comparative static analysis is a highly fruitful source of propositions which stand up quite well to empirical testing." We will show that durable structures given by history are relevant to the valuation of land and structures at a given point in time.

17 See empirical research by Dye and McMillen (2019) and by Munneke and Womack (2015) on the value of older structures that are reaching the point of redevelopment. The consequences of aging housing for equilibrium locations in AMM theory is beyond the scope of our analysis. Presumably owners anticipate the aging of their asset, as in real options theory, but how this maintains spatial equilibrium is an open question.

18 Throughout, parameters are given by lower case letters, variables by upper case.

$$H = aL + bS.\ a, b, L, S > 0 \tag{1}$$

The ability of structure to produce housing services increases with L and S according to the non-negative parameters a, b. The quantity of land L will be assumed fixed.

Equation (1) avoids a problem with the more typical Cobb-Douglas production function: the implausible assumption of constant land share in production. One observes flexible substitution in older suburban neighborhoods as well as in commercial real estate. Equation (1) delivers a plausible rate of substitution between land and structure in addition to solutions which avoid messy log transformations.

Rent per unit housing services H – and therefore value per unit – declines with the amount of structure as demonstrated by empirical studies summarize by Munneke and Womack (2015):

$$\text{Rent}/H = p\left(\frac{S^{-c}}{L}\right), with\, 0 < c < 1.. \tag{2}$$

Here p is net (of maintenance costs) rent per unit intensity where intensity is measured by the term in parentheses. The decline of rent per unit structure as structure size increases is increasing with c. The limits on c are necessary to obtain rents that increase at a decreasing rate as structure size S increases. Evidence from rental markets show that rents do not decline steeply with unit size: i.e., c is close to zero.

The value of this property, a quantity that might be inferred from observed sales prices is obtained by multiplying equations (1) by (2) and calculating the present value of net rents:

$$P(H)H = \left(\frac{p}{r}\right)a\,S^{-c} + \left(\frac{p}{r}\right)b\left(\frac{S^{1-c}}{L}\right) \tag{3}$$

Here, r is the discount rate relevant to calculating net present value. We simplify by assuming that rents and discount rates are unchanged into perpetuity: then $P(H) = (\text{Rent}/H)/r$.[19]

The cost to build a unit of structure is a constant dollar amount per unit structure:

$$\text{Building costs} = k\,S^d,\ \text{given}\ d, k > 0. \tag{4}$$

Here, k is the dollar cost to build a unit structure. If $0 < d < 1$ then building costs per unit structure decline with structure size; this is expected for one- or two-story structures. If $d > 1$, then our model is

[19] One cannot conclude that the first term of equation (3) is land value and the second structure value because optimized land value is a residual derived below.

relevant to larger structures that require increased structural strength and lifting building materials to higher levels (see Eriksen and Orlando, 2019). Moreover, $d>1$ provides a simple way of capturing additional costs such as basements and detached garages that typically accompany larger houses. The model solution can be simplified with $d=1$. We ignore demolition costs for simplicity and site preparation costs are modeled as part of vacant land value.

As if vacant land value

Land value as if vacant, the appraisal definition of urban land with an existing structure, is derived from highest and best use (HBU) which is the structure size S^* that will maximize the value of vacant land, V. After the existing structure has been demolished, the land owner will choose structure size S to maximize the land residual value, the difference between capitalized rental income from the structure and the cost of construction:[20]

$$\text{Max over } S\text{:}\quad P(H)H - kS^d \tag{5}$$

The maximized difference is land value at the HBU use, $V^{\square}$:

$$V^* = \left(\frac{p}{r}\right) a S^{(-c)} + \left(\frac{p}{r}\right) b \left(\frac{S^{(1-c)}}{L}\right) - kS^d. \tag{6}$$

Here, asterisks (*) signify optimized values. This is the land residual value at the point of reconstruction: i.e., after the existing structure becomes valueless and it has been demolished. It is a hypothetical ("as if") value because it is observed only at the point after the option to tear down has been exercised, i.e., after the existing structure is removed. We allow the removal to occur for a number of reasons such as natural disaster prior to the optimal teardown time.

First and second order conditions for maximization and other technical details are available from the authors on request.

The evolution of land and structure value over time

The meaning of real estate as a bundled good is rigorously defined by the model. After the new structure is completed, construction costs are sunk and irrelevant to valuation of the property which is given by equation (3). Changes over time are determined by changes in rents per unit services and the

[20] There is no option value in this certainty model, an assumption entirely consistent with our focus on new construction and property value in the years immediately after.

interest rate, $\frac{p}{r}$ which has proportional impact on both land and structure.[21] I.e., an important empirical implication is that land and structure will change at the same percentage rate after construction except that the structure may depreciate.

An exponential rate of depreciation, a simple approximation to reality, has been empirically documented by Bokhari and Geltner (2019) and is consistent with the first 30 years of structure life modeled by Goodman and Thibodeau (1995):

$$S_\delta = S^* e^{-\delta age}, \text{ with } 0 < \delta < 1. \tag{7}$$

Here δ is the annual rate of depreciation and age is structure age in years. The quantity of structure is given by S_δ. This setup gives the relationship between land value and structure value as the structure ages, and allows unexpected changes in rents or interest rates after construction:[22]

$$P(H_\delta) H_\delta = \left(\frac{p}{r}\right) a S^{-c} e^{c\delta age} + \left(\frac{p}{r}\right) b \left(\frac{S^{(1-c)} e^{-(1-c)\delta age}}{L}\right). \tag{8}$$

$$P(H_\delta) H_\delta = \left(\frac{p}{r}\right)\left[a S^{-c} + b\left(\frac{S^{(1-c)} e^{-\delta age}}{L}\right)\right] e^{c\delta age} \tag{9}$$

When the structure is new then $SV_0 = kS^d$, the cost to build. But as the structure depreciates it becomes a smaller percentage of property value because land value is not a function of depreciation. Equation (9) shows that property value – i.e., the bundle of land and structure – evolves over time as $\frac{p}{r}$ changes and that the quantity of structure changes according to equation (7). This implies that land value gradually increases as a ratio to property value after construction.

Empirical implications of the bundled good assumption are:

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21 Structure quantity enters both terms of equation (3) so we cannot conclude that the first term is land value and the second structure value: the two are bundled after construction to $S^{\square}$.

22 Equation (8) implies that c must be sufficiently close to zero to allow for property value to decrease with age. This is consistent with the empirical observation that rents on large apartments are only 5% to 10% below those on large apartments.

- Both land and structure value change proportionally with any change in rents or interest rates, $\frac{p}{r}$. The land value ratio as a function of $\frac{p}{r}$ is constant.
- As structure depreciates the land value ratio becomes larger. This is the only reason for a change in the land value ratio over time.
- The land value ratio for new construction is calculated from equation (6) divided by (8) (or equivalently (9)) given that $age=0$. Over time this ratio is adjusted with depreciation according to the back casting algorithm to be explained below. Economic structure value is calculated over time from equation (9) and the land value ratio.

The land residual model

The land residual model values depreciated structure at the cost to replace:

$$SV_{\delta,res}=k\,S^{d}\,e^{-d\delta age}. \tag{10}$$

Here $SV_{\delta,res}$ is the depreciated structure value given land residual assumptions. When the structure is new (age =0) this follows from the fact that the labor and materials to build must be bid away from alternative uses as in the AMM model. I.e., the land residual and bundled good assumptions agree when age =0 and the two models give the same structure value. Here the AMM assumptions hold because land and structure are traded separately at the time of construction. But once the structure begins to depreciate, the economic value of the structure calculated with bundled good assumptions can depart from depreciated cost to build, equation (10).

Land value under the land residual assumption is given by $LV_{\delta,res}$, the difference between property value and the replacement cost of the structure:

$$LV_{\delta,res}=\left[\left(\frac{p}{r}\right)a\,S^{-c}+\left(\frac{p}{r}\right)b\left(\frac{S^{(1-c)}e^{-\delta age}}{L}\right)\right]e^{c*age*\delta}-k\,S^{d}\,e^{-d\delta age} \tag{11}$$

Equations (10) and (11) motivate empirical implications from land residual theory. Construction costs k typically change slowly over time whereas rents and interest rates can be quite volatile. This implies that land residual value, equation (11) can be quite volatile relative to structure value. The two can move independently as construction costs and depreciation are not necessarily linked to changes in $\frac{p}{r}$. The empirical implications under land residual assumptions are:

1. Land residual structure values change only with construction costs and depreciation whereas land values changes are leveraged by construction costs. Any change in property value due to rents and interest rates is forced into land value by the model.
2. The land value ratio changes with prices, rents, construction costs and depreciation. Since changes in land value are leveraged by construction costs, the land value ratio may be much more volatile than the ratio under the bundled good model.
3. The land value ratio is calculated over time from equations (9) and (11).

These empirical implications can differ radically from those under the bundled good assumption, even during the first 10 or 15 years of structure life, if there are substantial changes in $\frac{p}{r}$.

A Hybrid Model: a component of land value can evolve separately from HBU structure value

Discussions with participants at the Lincoln Institute for Land Policy have produced some situations in which there can be some independence between land values and structure value. One example is waterfront locations which are much more valuable than locations just behind the waterfront houses. In this case the market may pay a premium purely for the location and independent of the substitution of structure for land. i.e., the property is more valuable and a bigger more expensive new house should be built, but there is also a waterfront premium that is not reflected in construction costs. Similarly, prestige value is associated with a neighborhood of large houses on large lots: a famous example is the 90210 zip code for Beverly Hills, California.

The second example is a corner lot on a busy street. The location is worth less than an interior location on a quiet street. A smaller, less expensive house will typically be built on the corner lot, but the lot may have some negative value independent of new construction decisions.

We model the partial independence of some location values by modifying equation (9) with a parameter that governs the premium and discount:

$$P(H_\delta)H_\delta=\left(\frac{p}{r}\right)\gamma\left[aS^{-c}+b\left(\frac{S^{(1-c)}e^{-\delta age}}{L}\right)\right]e^{c\delta age}+(\gamma-1)M. \tag{12}$$

Here, $\gamma\approx 1$ is the premium or discount to the value of net rental income in the market as a whole; most properties will have $\gamma=1$ and others will have pure land premiums or discounts over market average values, M.

We will implement the hybrid model by including neighborhood dummy variables in the AVM regression. The resulting land value ratios can be compared to those produced without neighborhood dummies.

Numerical examples with empirical estimates

Appendix Table 1 assumes some values for parameters and solves 6 for V*. Most parameters are chosen for convenience such as values of 1 for a and b. Others were chosen for values consistent with empirical evidence: c=0.05 and delta=0.03. When we set p/r = 18 and k=10, we found S* = 19, V*=156 and structure value (construction costs) = 255: the land value ratio for new construction is 38% which is close to our estimates from Maricopa data.

To simulate changes in p/r over the first 12 years of a structure life we use an actual house price index (HPI) for a Maricopa housing market (market #5) from 2007-2018, a period when Maricopa had one of the most volatile housing markets in the US with prices declining by 62% by 2011, then increasing by a similar amount by 2018. We combine the house price changes with our assumption of a 3% annual depreciation rate and the model land value ratio of 38% which is assumed to apply in 2007 to produce simulated indices of land value ratios under the two model assumptions. We converted the ratios to indices with values normalized to 1.0 in 2018 (Figure 1).[23]

– insert Figure 1 about here –

Under the bundled good assumption, the index of land value ratio changes from 0.81 to 1.0 over the 12-year period whereas the change under land residual assumptions are enormously volatile: model ratios (not indexed) in 2012 are .47 and .385. This volatility is not an artifact of our simple model; in fact, it follows from the high volatility of house prices in Maricopa combined with the land residual assumption that movements in structure value are governed by changes in construction costs (assumed to be 1% per year in our simulation) and depreciation (assumed 3% per year). Under these assumptions the land value ratio can be less than zero and greater than unity in some years: it was slightly negative in 2011.

[23] It is coincidental that house prices and residual ratios are close to 1.0 in 2007 as well as 2018, reflecting U-shaped Maricopa prices. The derivation of numbers for Figure 1 is clarified in Appendix Table A1, Panels B and C.

An algorithm for implementing theory

The "split tax algorithm" is all that is needed to separate land and structure value for tax purposes. The remainder of the algorithm is presented to determine if tax equity is increased by further modelling structure value.

Steps in the split tax algorithm:

1. Estimate a baseline hedonic model ("AVM") using sales data for new construction, where new construction can be defined as less than 15 years old; or some higher age limit can be set. The idea is to exclude properties with any significant value in the option to redevelop.
    a. Include annual time dummies in the AVM. The dummies have values of 1 for the year the sale was observed, otherwise zero. Calculate predicted values in the year of sale (AVM_syear) and in the year built (AVM_yrblt).
    b. Assessed value to sales price is avratio_AVM = AVM_syear/sprice_syear where sprice_syear is the observed sales price in the year of sale.
    c. Calculate baseline Coefficient of Dispersion (COD) from the assessment ratio, avratio_AVM.
2. Run loops to back cast cost to build and hedonic value, both estimated for year built ("yrblt"). This is done for the possibly limited sample which includes information on the cost to build in the year built and the property sold in that year or a future year. The structure must be considered "new" (i.e., with little value in the option to renovate or replace) when it is sold.
    a. Back cast cost with cost manual estimates for a current year (2018 in this study) combined with a local construction cost index to estimate cost_yrblt.
    b. Back cast sales prices with the coefficients on the time dummies from the AVM (estimated in step #1) to produce sprice_yrblt.
    c. Calculate land value in the year of construction: lv_yrblt = sprice_yrblt - cost_yrblt.
    d. Calculate the ratio of land value to total value in the year built: lvratio_yrblt= lv_yrblt/ AVM_yrblt.
    e. Update lvratio_yrblt to syear (=lvratio_syear) using an estimate of the rate of depreciation. We approximate with an exponential depreciaiton rate as discussed in the literature review section.
        i. The formula is: lvratio_syear = (sprice_yrblt - cost_yrblt*exp(-δage))/ AVM_yrblt, where δ is the annual rate of depreciation and age is structure age in years.

ii. This implements the main empirical prediction of the back casting model which is that depreciation is the only reason for changes in the land value ratio at the property level once the property starts trading as a bundled good.

3. Specify and estimate a land value ratio model as of the sale year, lvratio_syear.
   a. Model specification uses characteristics from the Step #1 AVM and follows theory, equations (1) – (9).
      i. The simplest theory says that there should be no influence of time or location characteristics: land value is constant across space within a market area and over time given that we have already controlled depreciation.
      ii. Hybrid theory, equation (12), says that year of sale and location characteristics do influence that part of land value which varies independently of property value.
   b. Additional specification may be based on the theory of substitution of structure value for land value – land characteristics matter:
      i. Small lots may limit the ability to build a structure that is HBU for the neighborhood. Expect a negative sign on small lot variables.
      ii. Large lots may contain "excess acreage," meaning that the lot is much bigger than required to build a HBU structure. Expect little addition to the lvratio for large lots, except that the possibility of subdividing into two or more lots may increase the lvratio.
   c. Use the coefficients from the lvratio model to predict "normalized" land value ratios, lvratio_syear_hat where normalization controls ideosyncratic characteristics associated with decisions to build and sell new properties.
4. Extrapolated predictions are to a possibly much larger set of observations of "new" structures (i.e., with little option value) where we did not have information on construction costs and sales prices. The extrapolation of land value ratios is from the possibly selective sample used in step #2 to different neighborhoods and older structures.[24] Extrapolation is essential for the purposes of tax assessment or valuing land and structure for underwriting and investment purposes because properties must be valued even if they did not sell.

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[24] By extrapolating, we maintain comparability to Davis, et al., 2019 who use a kriging method to extrapolate land values to properties more than 15 years old or otherwise out of sample. They exclude sales of properties they judge to have high option value, similar to our focus on relatively new structures.

Additional steps to model structure value and calculate CODs

5. Specify and estimate a hedonic model designed to model structure value in the sale year. This is necessary so that we can use CODs to compare valuations calculated from back casting to those based on AVM_syear/sprice_syear from Step #1.
    a. Calculate land value: lv_syear = lvratio_syear_hat*AVM_syear.
    b. Calculate structure price in the year of sale: struct_price_syear = sprice_syear – lv_syear.
    c. Estimate the model of structure value with struct_price_syear as the dependent variable and use the coefficients to predict struct_value_syear.
    d. Model structure value based on the same theory used to construct the AVM model in step #1.
6. Calculate the assessment ratio using the back casting method: avratio_back = (lv_syear + struct_value_syear)/sprice. This is motivated by AVM theory as well as industry practice: the value of new construction should be the sum of its parts.
7. Calculate COD's for avratio_back and avratio_AVM. Low values are preferred.

Back casting will be useful for assessment purposes if it produces lower CODs than avratio_AVM. Similarly, back casting can be compared to the land residual method as explained next.

Land residual algorithm compared to back casting

The land residual method begins with step #1, then simplifies by assuming that structure value can be obtained from the depreciated cost to build. Important differences between the two models as applied here:

- Land residual uses the cost to build in the year of sale whereas back casting uses cost in the year of new construction.

- Land residual depreciates cost in the year built whereas back casting depreciates cost in the year of construction. The percentage depreciation is the same in both methods, the exponential of the depreciation rate times structure age at time of sale.

- Most importantly, land residual calculates the land value ratio based on AVM value (step #1) at the date of sale whereas back casting calculates the ratio based on the AVM value at the time of construction. In a rapidly changing market, this can lead to very different ratios.

The most recent version of the land residual model (Davis, *et al.*, 2019) calculates land value ratios only for new construction, similar to back casting. Here we define "new" in the same way for both methods. We apply step #4 in the back casting algorithm to residual land value ratios: model specifications are identical to ensure that the resulting COD's can be compared.

4. Data and Results

Maricopa county assessor and GIS data

As part of the Lincoln land valuation project, the Maricopa county assessor provided several data files. Importantly, the assessor has divided the county into market areas and each market area is divided into neighborhood boundaries in recognition of the fact that values can differ substantially by neighborhood. All data contain latitude and longitude coordinates. Data described in Table 2 include:

- Improved property sales prices, dates, and characteristics as described below. We focus on single family residential (SFR) annual sales data for 2007 through 2018.
- For this report, we added horizontal and vertical location data based on GIS analysis. In addition, we add annual FHFA land value and land value ratio estimates from 2012-2018 for zip codes in Maricopa county. Finally, we add 2017 Maricopa assessed values by parcel.
- We calculated construction costs using a cost manual using Phoenix square foot cost multipliers, additional floor adjustments, cost of a basement, garage, outbuildings, swimming pool and sports courts. Costs are adjusted for construction quality information from the Maricopa assessor. Details are in Clapp et al. (2021).

Table 1 describes data filtering starting with all SFR sales in Maricopa county. We filter data to drop the top and bottom 1% for each variable, drop non-arm's length transactions and require complete data for all variables in the regression models. This reduces average sales price from about $545,000 to $487,000 but otherwise has little effect on average characteristics.

Market 5 is chosen as a good place to work out the complexities of land and structure valuation (e.g., to test "proof of concept" of back casting), not as a representative area: it has ample transactions for new construction whereas new construction transactions are scarce in many urban areas.[25] Market 5 is much higher priced than the average Maricopa market and it has larger, older houses on larger lots. We

[25] We chose market 5 as a good place to study all three "buckets" of land value: no option value (this paper), high option value and intermediate cases. Clapp et al. (2021) focus on the second and third buckets, using the relatively large numbers of teardown and vacant land sales in market 5. They provide more detail on the choice of market 5 as compared to four other market areas.

focus on new construction in two categories: structure built since 1999 compose our estimation sample with results extrapolated to those built in the 10 years from 1990 through 1999. Since sales data end in 2018, this gives houses that are no more than 28 years old at sale. In-sample estimation is for houses built since 1999 which are no more than 18 years old.

Table 2 describes data we calculated based on GIS analysis, and it includes explanatory variables relevant to the automated valuation model (AVM). Comparing the in-sample (built since 1999) to the extrapolation sample (built 1990 – 1999), average sales prices increased by over 25% for the later construction, suggesting that new construction was focused on the most desirable neighborhoods, and reflecting demand for larger structures on larger lots. This is expected in a rising market where houses are built to HBU: i.e., optimal structure sizes increased in response to changing demand for location, much of which took place during the boom before the start of our sales data in 2007.[26]

– Insert Table 1 and Table 2 about here --

Baseline AVM specification and results (Steps 1 and 2)

Table 3 contains alternative specifications for an AVM (the "computer assisted" part of a CAMA model) designed to implement Steps 1 and 2b in the back casting algorithm. This is a standard hedonic model with variables for structure, land and location characteristics. A difference between our model and a standard hedonic, other than those dictated by data availability (e.g., we do not have number of bedrooms or bathrooms) is that our model will be used to calculate COD's when we extrapolate to new properties where we do not have construction costs, a task requiring that we strongly avoid overfitting which will produce noise in the extrapolated ratio of valuations to sales prices. The waterfall nature of the algorithm – fitted values from the baseline AVM will be used to estimate land value ratios which in turn will be used estimate structure values – provides another reason for our emphasis on parsimonious specifications. All models are fit to new construction (built since 1999) because we focus on this segment of the housing market.

Table 3 contains several models designed to allow nonlinear effects of land and structure variables. We evaluated the robustness of coefficients across different specifications when comparing the most parsimonious specification, Model 1, to the others. The natural logarithm of interior area and lot area are the most economically and statistically important variables, followed by age, elevation and, in some

[26] Note that the distributions of sales year and construction quality are almost identical in the two samples, so differences in average price and property characteristics must reflect changes in location and HBU demand. This is reflected in changes in the percentage of properties built at relatively high elevation: 34% for construction since 1999 versus 10.5% for the earlier 10-year period.

specifications, the two distance variables. Focusing on these coefficients, the most striking finding is that they respond in plausible ways to the presence of neighborhood dummy variables. When neighborhood dummies are included, groups 2 and 3 are significant and group 3 is associated with a 0.38 or 0.39 increase in log house prices. Distance to the CBD becomes important with a positive sign (more peripheral locations are valued more) after introducing neighborhood dummies.[27] We conclude that the value of new construction is strongly dependent on where it is located and that adequately controlling location is important.

Property quality becomes more important after controlling location with neighborhood group dummies, and distance to the downtown (CBD) becomes large and significant. We conclude that the assessor drew neighborhood boundaries so as to isolate quality and that more peripheral locations are more valuable after controlling neighborhoods. Simple log specifications for interior area, lot size and structure age compare favorably to quadratic specifications for these variables. Models 4 and 5 have similar coefficients but Model 4 uses fewer variables without sacrificing explanatory power.[28] Also, the log variables are somewhat easier to interpret as elasticities. Therefore, we chose Model 4 as our preferred baseline specification.

Table 4 contains land value ratios where property value is estimated using Model 4, and a 3% depreciation rate is applied as discussed above. Back casting and land residuals have the same mean ratios, 0.54 and their distributions are similar. This is a surprising result given that theory (Figure 1) predicts much more volatility over time for the residual method. Appendix Table 2 shows that this is due to our sample with the U-shaped time pattern of house prices illustrated in Figure 1. The residual (back casting) land value ratio was 0.64 (0.53) in 2007, falling to 0.44 (0.54) in 2011, the bottom of the bust. By 2013 the two ratios were nearly identical 0.54 (0.55); in the next 5 years the residual ratio averaged about 0.02 higher than back casting. The overall average is identical despite much more volatility in the residual method, a volatility that is not compatible with different tax rates for land and structure, criterion 1.

– Insert Table 4 about here –

[27] The coefficient on the distance to the CBD variable is large in models 2, 4 and 5, but the maximum variation is only about .5 and for over 50% of properties the variation is only .25. This means that the influence on log of sales price is within about 10% for most properties.

[28] The coefficients on age variables in model 5 are consistent with 3% per year exponential depreciation through the first 18 years of structure life, a rate agreeing with the literature reviewed above. We will use 3% throughout.

Table 4 shows that the land value ratio is strongly positively associated with neighborhood value and that land residual and back casting ratios respond in the same way. The similarity between the two methods follows because all three neighborhood value groups have a U-shaped pattern of house prices. The strong positive association implies that land values increase faster than HBU construction as property values vary across space. This would not be the case if elasticity of substitution between land and structure were one. Table 4 provides support for the hybrid model: there is a portion of land value which increases with property value at a higher rate than the value of HBU construction. As suggested by the hybrid model, this would be the case if neighborhoods have prestige value or if high elevation, perhaps associated with a view or view-access, were valued separately from the value of the lot for HBU construction.

Land value ratio model specification and results, Step #3

The specification of a land value ratio model follows the logic of our theoretical model. In its simplest form, the model says that ratios at the time of construction are the same throughout the market area: unlike the hybrid model, the simple model does not provide for any variation across neighborhoods. At the individual property level, the only variation across time is from structure depreciation, and we have already included depreciation when we calculated land value ratios at the time of sale. At the market level land value ratios might vary over time if expectations change to provide for different HBU structure values as a ratio to land value. This will occur only if the elasticity of structure-land substitution differs from the widely accepted (by scholars of urban land use) value of one. In summary, in its simplest form, theory does not anticipate much variation in land value ratios over time or space.

This motivates Model 1 in Table 5 which omits year dummies and neighborhood dummies. We then add these variables one group at a time in Models 2–4 in order to evaluate the hybrid model. Appendix Table 3 reports the same regressions for the land residual model.

A striking feature of the coefficients on structure variables is that they become insignificant or change sign when compared to Table 3, Model 4 (repeated as the last column in Table 5). This is particularly true of the most economically and statistically significant variables, log of interior area and structure age. The small, insignificant coefficient on interior area in the land value ratio model suggest that the association of interior area with land value is the same as with structure value but with opposite sign: i.e., the two coefficients cancel each other in the land value ratio model.

– Insert Table 5 about here –

Coefficients on other structural characteristics (structure age, construction class and the presence of a pool) are significant with signs opposite to the baseline in the last column, as one would expect these variables are influencing the denominator of the land value ratio to a greater extent than the numerator.

Additional support for the back casting method is obtained from the lot and location coefficients (Table 5). When compared to the last column, these coefficients are of the same sign which we would expect if we are measuring land value divided by the sum of land and structure value.[29] The fact that we can account for up to 68% of the variation in the land value ratio supports our modeling approach.

Observing changes in R^2 statistics, we conclude that neighborhood dummies are more important than year dummies. We are concerned that year dummies reflect the influence of the GFC on land value ratios rather than the intended relationship between HBU structure value and property value, so we test models without year dummies. When we omit year dummies, we force the regression to choose a single land value ratio over time for a property with a given vector of characteristics: i.e., we average out the influence of the GFC and of the boom which followed.

We will now focus on our preferred specification: the full specification with all the variables from the baseline AVM, Model 4 in Table 5. This is the hybrid model (neighborhoomd dummies are included) with year dummies. All the variables used in the baseline are also included in Model 4. Our many tests of the alternative specifications show that they make little difference to land values estimated for the purposes of split taxation as evaluated with our criteria: i.e., Model 4 is robust.

Many coefficients for the land residual Model 5 have the same signs and significance as those for back casting, and this is also true when we compare the first three models with land residual ratios as the dependent: see Appendix 3. One notable exception is log of interior area, an important variable that has a much larger coefficient in Model 5 than Model 4. We conclude that the two models are comparable in terms of their ability to explain their different land value ratios. The very high R-squared for land residuals follows logically from the fact that the denominator in the ratio is the predicted value from the baseline AVM whereas back casting uses the predicted value in the year of construction. If depreciated construction costs in the year of sale were a constant share of property value, then land residual regression would have an R-squared equal to one.

[29] The magnitudes of the coefficients cannot be compared because the baseline model is in logs whereas the dependent variable in Table 5 is the ratio of levels.

Application to separate taxation of land and structure

In sample predicted land value ratios and land value estimates from the two methods are compared to total valuation from the baseline specification in Table 6.[30] This is all that is required to tax land and structure separately, i.e., split taxation, without changing CODs which are given by the baseline AVM specification. This use of land value ratio models is particularly compelling because the model can be applied without changing existing CAMA systems. The land value ratio are simply multiplied by CAMA values.

– Insert Table 6 about here –

Both back casting and land residual models produce reasonable results. Importantly, minimum land value ratios are well above zero even though we include the bust period in sales, as was true in the raw data (Table 4). We did not have a problem with negative ratios, likely because we used a carefully researched 3% depreciation for structures. We did see negative ratios with much lower depreciation, but low depreciation for new structures is not consistent with evidence we reviewed or with age coefficients presented in Table 3.

As expected, given similar coefficients in table 5, land value ratios estimated with back casting are similar to land residuals. The main difference is that land residual values have somewhat higher variation than back casting. Both methods give a reasonable range of land value ratios, with half the ratios clustered in a range from about 0.5 to 0.6 for back casting, 0.47 to 0.62 for land residuals. This is desirable for separate taxation of land and structure since tax payers are likely to appeal assessments with large differences. Tables 3 and 5 show that much of the differences can be explained by neighborhood. This is desirable as long as the boundaries can be explained to tax payers.

Are the two methods equally effective for land value taxation? Figure 2 compares the land value ratios over time using the yearly coefficients from the models in Table 5. The figure provides empirical implementation of theoretical results in Figure 1: i.e., changes over time in both figures are driven by property value changes represented by the house price index. An important difference in the figures is that theory (Figure 1) tracks the value of an individual property with structure built in 2007 whereas Figure 2 is based on the aggregate of newly constructed properties, including those built before and after 2007.

– Insert Figure 2 about here –

[30] We only compare out preferred specification, Table 5, models 4 and 5. Other models give similar results.

Figure 2 shows little change over time in the back casting land value ratio which declined slightly during the GFC, then increased slightly and roughly linearly since 2011. These results support the model which says that there should be little variation overtime other than some increase due to structure depreciation.[31] Figure 2 shows that the land value ratio estimated by the residual model is much more responsive to the cycle than the hybrid back casting model.

We conclude that it is undesirable to value land and structure over time with the land residual model. The independent variation over a cycle is contrary to AMM theory where values are driven by net rents. Land value ratios calculated with the residual method will provide inaccurate descriptions of housing market risk. Most importantly, it is undesirable for property tax equity to shift dramatically over the housing market cycle as it will if the residual method is used. For example, the land residual model implies much more volatility in tax burden for those in high land value ratio neighborhoods. This will be difficult to explain to these tax payers, a violation of our criterion 1.

Robustness to counterfactuals and to fixed ratios

Table 6 compares to a counterfactual in which the back casting land value ratios are randomly reshuffled: i.e., the set of 2,140 ratios is randomly assigned to the set of explanatory variables. The results are averages from 100 runs, each with a different random shuffling.[32] We introduce the counterfactual to test the validity of the other two models here and when we extrapolate and evaluate with CODs. We further test robustness to a common method used by assessors, a fixed land value ratio which is represented here by the mean back casting ratio.

The ratios from back casting and land residuals are reasonable in the sense that they capture plausible variation across the 2,140 in-sample sales, whereas counterfactuals and fixed ratios do not. Clearly random ratios could not be explained to taxpayers; they would be perceived as unfair. But this is not true of fixed ratios, so one might ask "what have our models accomplished?" Ideally, we could now estimate equity measures, CODs that would differentiate models from counterfactuals. But CODs are determined from the baseline AVM, the second row of Table 6, not from the split between land and structure. This leads us to evaluate extrapolated ratios instead of CODs.

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[31] The small decline in the land value ratio during the GFC gives some support to the hybrid model: a portion of land value varies with market prices independently of HBU structure value.

[32] For each run, we estimated the Model 4, Table 5, then used the results to the means (e.g., mean of minimum values) reported in Table 6. Of course, the average coefficients for the land value ratio models are zero except for the constant which is .53, near the mean value for back casting ratios. Therefore, we do not present coefficients.

Extrapolating land value ratios (step 4)

Table 7 extrapolates land value ratios to the sales of structures built in the 1990s. Extrapolation is an important part of our "three buckets" strategy. Extrapolation is designed to show the relevance of a subsample of newly constructed properties to all properties that have not experienced obsolescence.

– Insert Table 7 about here –

The most striking information in Table 7 is that the mean baseline AVM value is 27% higher for properties built after 1999 than for those built in the 10 earlier years, \$683,000 versus \$537,000. This is remarkable because the distribution of sales years is the same for both sets of data (see Table 2, next to last line). This is not an artifact of the model (e.g., of age coefficients) because it holds for raw sales prices as it must given that the regression passes through mean values.[33] We ask whether these differences influenced our extrapolation results.

The extrapolated sample has somewhat higher land value ratios: 4% for back casting and over 6% for land residuals. But this is to be expected if aging is reducing the value of structure, as it will since we are extrapolating the positive age coefficients (+0.032 and +0.037, Table 5) from properties that have an average year built of 2005 (in-sample) to an average of 1995 (extrapolated sample).

Differences in age validate extrapolation, but we are concerned about differences in location. Neighborhood coefficients for the estimation sample (2,140 observations) may not be the best way to extrapolate to nearly five times as many sales, and this may explain some of the discrepancy in sales prices. Comparing distributions of sales prices by neighborhood (not shown), we find that in-sample and extrapolated-sample have nearly identical medians in the low valued neighborhood (\$485,000 vs. \$465,000) although we find bigger differences in the same direction occur in the tails of the distribution. The big differences in median sales prices (up to nearly 40%) occur in the higher valued neighborhoods. This may be related to a much higher rate of sales at high elevation in properties built after 1999: 34% vs. 11% (Table 2). Perhaps the positive coefficient on high elevation dummy (0.03 for back casting and 0.07 for land residuals) does not adequately capture differences in location.

In summary, application of our in-sample estimates to an extrapolation sample produces reasonable results, but so do counterfactuals, just as in Table 6. Our concern with differences between extrapolation and in-sample requires much more evaluation and adjustment. Any new construction sample may be influenced by selection on location and year of construction. However, further analysis of sample

[33] Small differences in mean values for sales prices vs. predicted prices are due to exponentiation of predicted log prices.

selection must be left to future research because this study set out to evaluate the back casting and land residuals in terms of criterion 4, their ability to support a structure value model that might reduce CODs in-sample, and this occupied much of our effort.[34]

Using external data to evaluate AVM and construction cost estimates

There are two components to the back casting model: 1) specifications of the AVM (i.e. the hedonic regression model) and 2) estimation of the cost to build a structure at the date of construction. These two components also hold for land residuals but both are evaluated at the date of sale, not date of construction. This section uses additional data to test the robustness of both of these components.

First, we use Maricopa County assessed values for 2017 to test robustness of our AVM specifications.[35] The assessor informs us that these values were official as of January 1, 2016 and that they are based on the sales of properties over the previous 2 to 5 years.

Table 8 reports results for all SFR sales and for sales of new construction over relevant time periods. We found high correlation coefficients, ranging from 0.82 to 0.92, between values obtained with our AVM and Maricopa assessed values. We conclude that our AVM is producing reasonable estimates of property value.

– Insert Table 8 about here –

However, COD's estimated with the assessor data or substantially and significantly below those based on our AVM. Discussion with the assessor revealed that the difference in COD's resulted from their use of much more detailed spatial data, including data on individual tract developments.[36] As discussed above, we went in the opposite direction by simplifying the 28 neighborhood identifiers provided to us in order to avoid over fitting. We are interested in extrapolating our results beyond the sample used to estimate parameters, so it is appropriate for us to avoid overfitting. By the way of contrast, the assessor is interested in reducing in-sample CODs in order to meet established standards.

[34] We have done preliminary work to test the validity of land value ratio models. These point towards future research designed to better control neighborhood and location in the baseline model.

[35] We are grateful to Professor Jeffrey Cohen for obtaining and sharing these data.

[36] They say: "both sets of values are explaining most of the same features, however ours has a little extra detail to account for additional variations in sales prices related to specific subdivisions/developments." (Source: May 10, 2021 email from the Maricopa Assessor's office.) This confirms our observations based on Google Street View. In addition, the assessor confirmed that CODs we calculate with their assessed values are similar to those they calculated.

Our AVM as applied to new construction (2,140 sales built since 1999) produced lower CODs than our AVM applied to all sales through 2015. The reduction from 0.134 to 0.101 is economically and statistically significant (Table 8). Moreover, the AVM CODs for 2 years of sales are nearly identical to 4 years of sales, as are Maricopa CODs. We conclude that: 1) changes in the AVM COD's for different samples reveal information relevant to assessment practice even though the levels of our COD's are considerably higher than those calculated with the additional spatial data used by the assessor; 2) we need further evaluation of location and specific subdivisions.

The second major component of the back casting model is estimation of construction costs in the year of sale. To evaluate this, we compared our construction costs and land value ratios to annual ZIP Code averages, 2012 through 2018, published by Davis at all (2019). Their depreciated cost to build in the year of sale are from professional appraisals conducted for the purposes of loan underwriting. We were able to merge 34 ZIP Code years (in seven ZIP Codes) where we had at least five transactions in our data. We compared the FHFA data with median values from our data in each ZIP Code year, and with median Maricopa assessed values discussed in the previous section.

– Insert Table 9 about here –

Table 9 shows high correlation between AVM values and property values from the FHFA data. The correlation between FHFA values and Maricopa assessor values is substantially lower whereas the AVM has a high correlation with assessor values at the ZIP code level (Model 3). AVM and FHFA structure values are related with an R-square of 0.71 (Table 9 Model 4). We conclude that our construction cost estimates are validated.

We obtain much lower correlations for land value ratios. Our reading of the technical notes provided by Davis *et al* (2019) suggested this is because of the methods they used to allocate land value ratios across space. We use actual sales data in market five which has much higher land and property values than the countywide sample Davis *et al* (2019) used with kriging to spread land value ratios to zip codes.

Can land and structure models reduce CODs? Structure value model specification and results, Step #5

As discussed in the introduction, models of structure value are required to assess CODs based on separating land and structure. As the last model in the waterfall structure of our algorithm, structure value models would appear particularly simple. Variables influencing land value have been used in steps #4a and #4b when calculating land value and structure price (sales price minus land value). Therefore, we expect structure value to be influenced by structure characteristics alone. We test this simple specification against a full specification which includes all variables in the baseline AVM, and against inclusion of time and neighborhood dummies.

– Insert Table 10 about here –

Table 10 Models 1 and 2 show that structure variables are of the same sign and significance as in the baseline AVM, as expected if we have isolated structure value.[37] Further evidence is obtained from the location variables (golf, elevation, etc.): none are significant. Lot size has a small positive association with structure value, as might be expected if more structure can be built on a larger lot. Omitting location characteristics has little influence on explanatory power (not shown): therefore, our preferred structure value model is the full specification which uses the same variables as the earlier steps in the waterfall.

The last two models in Table 10 are designed to check our suspicion that the structure coefficients are an artifact of the waterfall structure of our algorithm. Do back casting and land residual models perform better than a counterfactual or any other model? The answer is that they do not. Most tellingly, counterfactual random shuffling of land value ratios (Model 4) produces coefficients which are generally of the same signs and significance as the two models being tested. Clearly, these coefficient values come from the baseline model, not from information added by the land value ratio models. Comparison to a fixed land value ratio equal to the mean value of .54 (Model 5) provides compelling evidence that coefficient values are derived from the baseline AVM: our models of land value ratios add nothing to structure valuation. We find further evidence (not shown) against the structure models when we estimate structure value models for the extrapolation sample, Appendix Table 4.

Back casting and land residual COD's compared to the baseline AVM (steps 6 and 7)

The main result in Table 11 is that the additive model – property value equals land value plus structure value – in-sample COD (0.128) is much higher than the baseline (0.118). The difference of 0.01 is enormous to a tax assessor and it is statistically significant in the sense that the baseline confidence interval excludes the additive model point estimate and *vice versa*.[38] Land residual CODs are somewhat

[37] We cannot compare coefficient sizes because the dependent is estimated levels of structure prices in dollars versus the log of sales prices in the baseline AVM.

[38] The two confidence intervals overlap by 0.0018 on a range of 0.012, which we interpret as showing statistically significant differences. Appendix Table 5 shows that several subsamples produce differences in the 0.007 to 0.012 range, establishing the robustness of results in Table 11.

worse than back casting, but only by between 1 and 2%, a result that is not statistically significant. Our finding that the models do not improve assessment equity is compelling because the in-sample structures are relatively new and we estimate construction costs reliably as evaluated with FHA data.

Not surprisingly, extrapolating land values to the sample built after 1989 produces even larger CODs relative to baseline: the increase is marginally significant to assessors +0.004 (=0.132-0.128) but not statistically significant.

– Insert Table 11 about here –

We further test robustness with the fixed land value ratio model, which produced CODs very similar to back casting and land residuals. We find similar results with random reshuffling of land value ratios: one random draw is illustrated in Table 11. Our analysis above of the structure model coefficients points to a problem inherent in any such model: estimates of structure value are logically related to estimates of land value, here through the waterfall structure of the algorithm. This implies that separate estimates of the two values for extrapolation and CODs will necessarily add noise, reducing property tax equity. We conclude that it will be difficult to model both land and structure value so as to reduce CODs.

5. Summary, discussion of results and future research

This paper confronts the enormous complexity of estimating land value as a percentage of property value with criteria relevant to property tax assessment. Generally accepted theory says that new construction will typically maximize property value (i.e., it is highest and best use, HBU), implying that structure value can be estimated with construction cost. The back casting algorithm estimates the land value ratio with construction costs and property value, both estimated at the time of construction. After construction we assume that land and structure trade as a bundled good, implying that the ratio of land and structure value changes only with depreciation: that is, both land and structure respond to changes in property value in the same direction. We contrast this with land residual assumptions which allow structure and land value to evolve independently after construction.

We use four criteria to evaluate back casting and land residuals with Maricopa, Arizona, data on new construction since 1989 of single-family residential sales since 2006. Models of land value ratios produce coefficients consistent with separate identification of land and structure values. They support a hybrid of back casting and land residual theory: i.e., models with and without neighborhood controls suggest that there is some independent variation of land and structure value and this is confirmed by a strong positive association between property values and land value ratios across neighborhoods sorted from low to high value. If structure were easily substituted for land, an assumption maintained by much of the literature, then we should observe much less change across space in land value ratios for new HBU construction. Extrapolation of land value ratios to a much larger sample of houses averaging ten years older than the estimation sample produced reasonable results but raised questions about how to adjust for changing locations of new construction over time.

Findings for the land residual method are similar, but land value ratios are volatile over time, an artifact of that model which forces most variation in property value onto the land value component. This violates one of the criteria relevant to tax assessment of land and structure, namely that valuation equity should be easily explained and justified to tax payers.[39] We are unable to differentiate land value ratios calculate with either method (bundled or residual) from a counterfactual. This points towards future research into better modeling the selection of locations for new construction and to evaluation criteria focused on separate taxation of land and structures.

We evaluated and rejected a hypothesis that structure values could be modeled separately from land value ratios, and that the sum of structure and land values could reduce coefficients of dispersion (CODs), a standard tool for measuring property tax equity. Counterfactual analysis indicate that it is unlikely that any such effort will be successful because structure and land valuation are inherently interdependent: the dependent variable in a model of structure value is derived from a model of land value, implying that a structure model might add noise to CODs. Alternatively, future research into the relationship between location and land value ratios might produce a way to separately value structures.

Future research

Much of our effort went into disproving the idea that separate land and structure values would improve assessment equity. Now that data and logic reject this idea, research is free to focus on improving models of land value ratios, so that they can be used for split taxation by simply multiplying estimated ratios by CAMA property valuation.

A workable model of split taxation requires additional research in several areas:

1. Our results show that construction shifted to higher valued neighborhoods in the 2000's, suggesting the need for better controls for variation in land value ratios over space and time. Selective locations for new construction is a difficult issue for most land valuation models, requiring more research.

[39] Homes sold a few years apart might produce large changes (after revaluation) in land value taxes under land residual assumptions, with changes differing across neighborhoods.

2. Research into methods for evaluating the accuracy of land value ratios out of sample where accuracy is measured by property tax equity (i.e., CODs modified for split taxation). Such methods might be able to clearly distinguish different models of land value ratios from counterfactuals.

3. Much more work on the "three buckets" approach. This paper has focused on the first bucket, newly constructed properties defined as not having much value in the option to renovate or redevelop. It appears feasible to develop practical assessment tools for this bucket. More work is needed on the second bucket (some option value but not near the point of redevelopment) and third bucket (substantial option value).

4. More attention is needed to the case where property values are near or below cost to rebuild the existing structure. This is common in areas with depressed housing markets. Our model suggests that location still has substantial value in most such markets even though there will be little new construction, but more research is needed.

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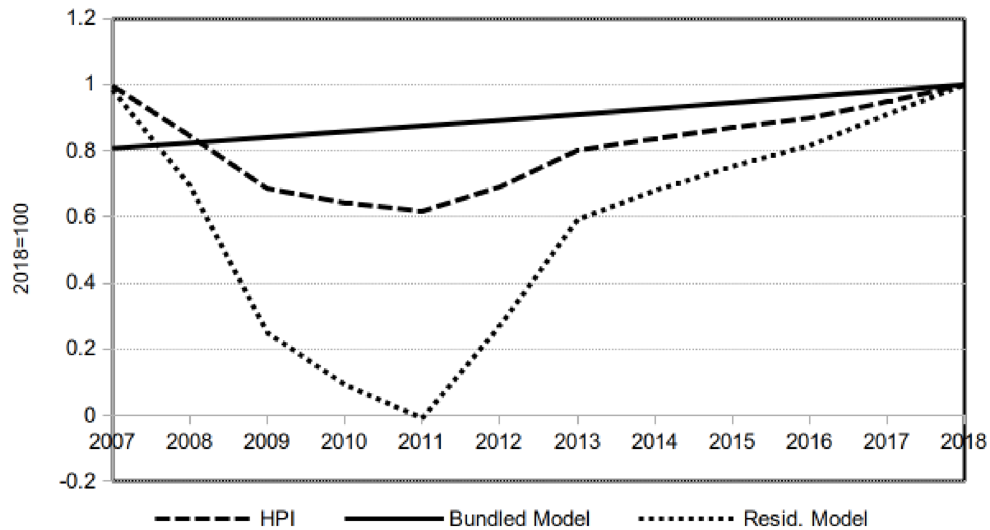
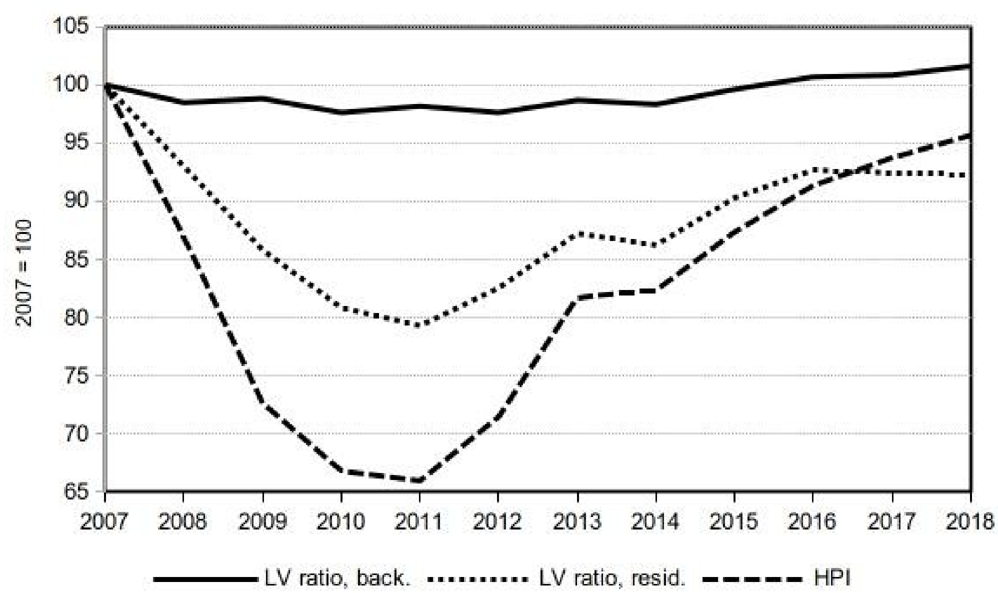
FIGURES

Figure 1: Land value ratios obtained from simple model and house price index



Notes: The market 5 house price index (HPI) plots the year coefficients from Table 3, Model 4. The numbers for land value ratios are from a numerical solution to the simple model. See the text and Appendix A1 for details.

Figure 2: House prices and land value ratios, 2007--2018



Notes: HPI is based on coefficients for year dummies from Table 3, Model 4. The back casted LV ratios are from year coefficients from Table 5, Model 4. The year coefficients for the residual LV ratios are based on Table 5, Model 5.

Table 1: Sample selection and summary statistics for key variables

	mean	*SD*	*p25*	*p50*	*p75*
(a) All Maricopa SFR sales, N=952,087					
price	310,185	1,055,229	143,500	217,000	315,917
year of sale	2013	3	2010	2013	2016
int. area (sqft)	1,983	943	1,425	1,822	2,378
lot size (sqft)	10,659	21,752	6,035	7,401	9,474
year built	1992	18	1980	1998	2005
(b) Market 5, N=42,410					
price	544,665	432,068	319,900	429,900	635,000
year of sale	2013	3	2010	2013	2016
int. area (sqft)	2,502	1,076	1,824	2,282	2,932
lot size (sqft)	14,103	13,528	7,423	10,106	14,782
year built	1984	12	1978	1985	1994
(c) Remove non-armslength transactions and outliers, N=36,555					
price	487,058	236,830	325,000	425,000	590,000
year of sale	2013	3	2010	2013	2016
int. area (sqft)	2,414	720	1,874	2,285	2,837
lot size (sqft)	13.06654	9.341162	7.734	10.106	13.993
year built	1984	12	1977	1984	1993
(d) Extrapolation sample, built 1990-1999, N=12,145					
price	543,746	227,102	380,000	489,000	665,000
year of sale	2013	3	2010	2013	2016
int. area (sqft)	2.646	0.778	2.020	2.548	3.205
lot size (sqft)	10,477	7,258	6,426	8,459	11,301
year built	1995	3	1992	1994	1997
(e) Sample built 2000-2018, N=2,140					
price	691,805	285,699	485,000	629,500	849,450
year of sale	2013	3	2010	2013	2016
int. area (sqft)	3,001	758	2,369	2,936	3,579
lot size (sqft)	12,977	10,184	7,025	9,981	13,585
year built	2005	5	2001	2003	2006

Notes: Panel (a) summarizes the sales data at MSA level while Panel (b) presents the corresponding statistics at the submarket level. We then drop the top and bottom 1% for each variable, drop non-arm's length transactions and require complete data for all variables in the regression models, Panel (c). This reduces average sales price from about \$545,000 to \$487,000 but otherwise has little effect on average characteristics. Panels (d) and (e) further subsample based on year of construction.

Table 2: Market 5 summary statistics for regression variables

	Built 2000–2018 (N=2,140)					*Built 1990–1999: extrapolation sample (N=10,005)*				
	mean	*SD*	*p25*	*p50*	*p75*	*mean*	*SD*	*p25*	*p50*	*p75*
price	691,805	285,699	485,000	629,500	849,450	543,746	227,102	380,000	489,000	665,000
price, fitted	682,756	255,620	485,863	650,512	836,079	537,120	200,353	383,890	488,624	663,694
int. Area (000' sqft)	3.00	0.76	2.37	2.94	3.58	2.65	0.78	2.02	2.55	3.21
lot size (000' sqft)	12.98	10.18	7.02	9.98	13.59	10.48	7.26	6.43	8.46	11.30
year built	2004.6	5.0	2001.0	2003.0	2006.0	1994.6	2.8	1992.0	1994.0	1997.0
ln(int. area)	7.97	0.26	7.77	7.98	8.18	7.84	0.30	7.61	7.84	8.07
ln(prop. age)	1.96	0.88	1.61	2.30	2.64	2.93	0.25	2.77	2.94	3.14
prop. quality	4.48	0.64	4.00	4.00	5.00	4.39	0.56	4.00	4.00	5.00
has pool	0.57	0.50	0.00	1.00	1.00	0.60	0.49	0.00	1.00	1.00
ln(lot size)	9.25	0.63	8.86	9.21	9.52	9.11	0.49	8.77	9.04	9.33
close to golf course	0.04	0.19	0.00	0.00	0.00	0.04	0.20	0.00	0.00	0.00
high elevation	0.34	0.47	0.00	0.00	1.00	0.11	0.31	0.00	0.00	0.00
ln(dist. water)	8.38	0.56	8.22	8.70	8.70	8.01	0.84	7.68	8.26	8.70
ln(dist. CBD)	3.43	0.13	3.36	3.47	3.52	3.41	0.11	3.35	3.44	3.48
year of sale	2012.9	3.4	2010.0	2013.0	2016.0	2013.0	3.4	2010.0	2013.0	2016.0
nbhd. group	1.87	0.59	2.00	2.00	2.00	1.89	0.61	2.00	2.00	2.00

Notes: This table presents summary statistics for the data we calculated based on GIS analysis, and it includes explanatory variables relevant to the automated valuation model (AVM). Comparing the in-sample (built since 1999) to the extrapolation sample (built 1990–1999), average sales prices increased by over 25% for the later construction, suggesting that new construction was focused on the most desirable neighborhoods, and reflecting demand for larger structures on larger lots. This is expected in a rising market where houses are built to HBU: e.g., optimal structure sizes increased in response to changing demand for location, much of which took place during the boom before the start of our sales data in 2007. Mean fitted sales price differs slightly from mean sales price because log of sales price is the dependent in the AVM.

Table 3: Hedonic price regression estimates (AVM's)

	(1)	*(2)*	*(3)*	*(4)*	*(5)*
	ln(price)	*ln(price)*	*ln(price)*	*ln(price)*	*ln(price)*
ln(int. area)	0.635***	0.580***	0.674***	0.617***	
	(8.57)	(9.35)	(10.06)	(10.47)	
ln(prop. age)	-0.078***	-0.088***	-0.067***	-0.078***	
	(-4.19)	(-6.83)	(-3.64)	(-7.04)	
prop. quality=4	-0.023	0.083	-0.030	0.068	0.054
	(-0.26)	(0.88)	(-0.45)	(0.96)	(0.72)
prop. quality=5	0.021	0.155	0.030	0.155**	0.148**
	(0.21)	(1.64)	(0.38)	(2.24)	(2.06)
prop. quality=6	0.034	0.202**	0.024	0.184***	0.143*
	(0.32)	(2.27)	(0.28)	(2.80)	(2.05)
has pool	0.059***	0.059***	0.046***	0.046***	0.046***
	(5.17)	(5.30)	(3.92)	(4.53)	(4.70)
ln(lot size)	0.235***	0.208***	0.218***	0.192***	
	(9.04)	(8.61)	(8.16)	(8.63)	
close to golf course	0.188***	0.101**	0.190**	0.104***	0.107***
	(2.78)	(2.45)	(2.66)	(2.82)	(2.96)
high elevation	0.158***	0.140***	0.176***	0.155***	0.158***
	(3.46)	(3.32)	(4.75)	(4.72)	(4.33)
ln(dist. water)	-0.090**	-0.038*	-0.103***	-0.051***	-0.052***
	(-2.48)	(-1.85)	(-3.07)	(-2.94)	(-3.04)
ln(dist. CBD)	-0.014	0.371***	0.005	0.386***	0.475***
	(-0.06)	(3.81)	(0.02)	(3.59)	(3.84)
nbhd. group=2		0.024		0.033	0.034
		(0.84)		(1.49)	(1.46)
nbhd. group=3		0.380***		0.380***	0.392***
		(5.71)		(5.96)	(6.01)
year dummies	No	No	Yes	Yes	Yes
int. floor area (1,000 sqft.)					0.355**
					(2.60)
int. floor area, squared					-0.022
					(-1.07)
prop. age, 100 years					-1.710***
					(-3.94)

prop. age, squared					2.480
					(1.44)
lot size (10,000 sqft.)					0.266***
					(9.95)
lot size, squared					-0.036***
					(-7.84)
Constant	6.989***	5.775***	7.057***	5.857***	11.119***
	(9.91)	(16.16)	(10.35)	(17.12)	(30.14)
Observations	2140	2140	2140	2140	2140
R^2	0.681	0.737	0.791	0.845	0.844

Notes: t statistics in parentheses use robust standard errors, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Models 1–5 present alternative specifications for an AVM (the "computer assisted" part of a CAMA model) designed to implement Steps 1 and 2b in the back casting algorithm. These are standard hedonic models with variables for structure, land and location characteristics. A difference between our model and a standard hedonic, other than those dictated by data availability (e.g., we do not have number of bedrooms or bathrooms) is that our model will be used to calculate COD's when we extrapolate to new properties where we do not have construction costs, a task requiring that we avoid overfitting which will produce noise in the extrapolated ratio of valuations to sales prices.

Table 4: Price and LV ratio estimates, by neighborhood group

Variable	*mean*	*min*	*p25*	*p50*	*p75*	*max*
			(a) Neighborhood group 1, N=520			
price	564,033	121,500	369,495	485,000	712,500	1,400,000
LV ratio, back casted	0.46	0.24	0.36	0.47	0.54	0.72
LV ratio, resid. method	0.45	0.24	0.36	0.45	0.55	0.68
			(b) Neighborhood group 2, N=1,370			
price	697,559	215,250	515,000	640,000	822,500	1,565,000
LV ratio, back casted	0.55	0.30	0.51	0.55	0.60	0.74
LV ratio, resid. method	0.55	0.18	0.50	0.56	0.63	0.76
			(c) Neighborhood group 3, N=250			
price	926,036	135,000	650,000	874,000	1,200,000	1,565,000
LV ratio, back casted	0.65	0.46	0.60	0.64	0.69	0.82
LV ratio, resid. method	0.64	0.20	0.60	0.64	0.69	0.77
			(d) Total, N=2,140			
price	691,805	121,500	485,000	629,500	849,450	1,565,000
LV ratio, back casted	0.54	0.24	0.49	0.55	0.61	0.82
LV ratio, resid. method	0.54	0.18	0.47	0.56	0.62	0.77

Notes: This table summarizes land value ratios where property value is estimated using Table 3, Model 4, and a 3% depreciation rate is applied. This is the result of steps #1 and #2 of the back casting algorithm, before modelling the ratios. Land residual methods differ by estimating depreciated cost as of the year sold, and dividing by AVM value in the sales year. Back casting and land residuals have the same mean ratios, 0.54 and their distributions are similar. The LV ratios rise with property values: they are lowest in neigborhood group 1 and highest in the most affluent submarket, neighborhood group 3.

Table 5: Back casted LV ratio regression estimates

	(1)	*(2)*	*(3)*	*(4)*	*(5)*	*Table 3M4*
	LV ratio, back.	*LV ratio, back.*	*LV ratio, back.*	*LV ratio, back.*	*LV ratio, resid. method*	*ln(price)*
ln(int. area)	0.020	-0.005	0.021	-0.005	-0.099***	0.617***
	(0.93)	(-0.25)	(0.97)	(-0.26)	(-11.80)	(10.47)
ln(prop. age)	0.040***	0.034***	0.038***	0.032***	0.037***	-0.078***
	(6.41)	(8.14)	(6.00)	(7.60)	(14.81)	(-7.04)
prop. quality=4	-0.144***	-0.101***	-0.141***	-0.100***	-0.115***	0.068
	(-6.89)	(-5.87)	(-5.92)	(-6.16)	(-15.55)	(0.96)
prop. quality=5	-0.218***	-0.167***	-0.213***	-0.164***	-0.148***	0.155**
	(-9.01)	(-9.17)	(-7.96)	(-9.39)	(-17.24)	(2.24)
prop. quality=6	-0.305***	-0.232***	-0.301***	-0.230***	-0.198***	0.184***
	(-8.79)	(-11.21)	(-8.09)	(-11.09)	(-17.67)	(2.80)
has pool	-0.022**	-0.021***	-0.023**	-0.022***	-0.014***	0.046***
	(-2.62)	(-3.39)	(-2.47)	(-3.29)	(-7.01)	(4.53)
ln(lot size)	0.071***	0.059***	0.069***	0.058***	0.068***	0.192***
	(5.16)	(6.25)	(5.16)	(6.10)	(14.39)	(8.63)
close to golf course	0.050	0.012	0.052	0.015	0.059***	0.104***
	(1.21)	(0.85)	(1.26)	(1.04)	(11.80)	(2.82)
high elevation	0.039***	0.026***	0.043***	0.029***	0.068***	0.155***
	(3.49)	(3.30)	(3.86)	(3.43)	(38.96)	(4.72)
ln(dist. water)	-0.029	-0.005	-0.031	-0.006	-0.028***	-0.051***
	(-1.48)	(-0.41)	(-1.59)	(-0.55)	(-6.29)	(-2.94)
ln(dist. CBD)	0.098	0.242***	0.099	0.245***	0.099***	0.386***
	(0.95)	(4.96)	(0.95)	(4.75)	(4.72)	(3.59)
nbhd. group=2		0.031*		0.032	0.043***	0.033
		(1.87)		(1.68)	(8.15)	(1.49)
nbhd. group=3		0.175***		0.175***	0.135***	0.380***
		(11.18)		(10.94)	(22.76)	(5.96)
Year dummies	No	No	Yes	Yes	Yes	Yes
Constant	-0.269	-0.728***	-0.240	-0.707**	0.707***	5.857***
	(-0.61)	(-2.97)	(-0.54)	(-2.74)	(13.03)	(17.12)
Observations	2,140	2,140	2,140	2,140	2,140	2,140
R^2	0.474	0.665	0.492	0.682	0.933	0.845

Notes: t statistics in parentheses use robust standard errors, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. This table presents estimated coefficients from a set of regressions that model back casted LV ratios. In its simplest form, theory does not anticipate much variation in land value ratios over time or space, which is why Model 1 omits year dummies and neighborhood dummies. We then add these variables, one group at a time, in Models 2–4 in order to evaluate the hybrid model. A striking feature of the coefficients on structure variables is that they become insignificant or change sign when compared to Table 3, Model 4 (repeated as the last column). This is particularly true of the most economically and statistically significant variables, log of interior area and structure age. The small, insignificant coefficient on interior area in the land value ratio model suggest that the association of interior area with land value is the same as with structure value but with opposite sign: i.e., the two coefficients cancel each other in the land value ratio model. Coefficients on other structural characteristics are significant with signs opposite to the baseline in the last column, as one would expect if these variables are influencing the denominator of the land value ratio to a greater extent than the numerator. Analogously, coefficient on lot related hedonics do not exhibit opposite signs to the baseline price model.

Table 6: Fitted values for sales prices, LV ratios and land values, built 2000–2018

Variable	*N*	*mean*	*min*	*p25*	*p50*	*p75*	*max*
Price	2,140	691,804	121,500	485,000	629,500	849,450	1,565,000
Price, fitted	2,140	682,756	218,605	485,863	650,512	836,079	1,959,172
(a) Based on back casting							
LV ratio, back casted	2,140	0.54	0.32	0.50	0.54	0.59	0.81
Land value, back casted	2,140	375,714	96,886	261,249	350,014	468,452	1,316,292
(b) Based on residual method							
LV ratio, resid. method	2,140	0.54	0.27	0.47	0.55	0.62	0.83
Land value, resid. method	2,140	380,177	85,491	250,142	348,670	483,482	1,364,374
(c) Based on back casting, reshuffled LV ratios							
LV ratio, back casted and reshuffled	2,140	0.54	0.50	0.53	0.54	0.55	0.57
Land value, reshuffled LV ratios	2,140	368,219	118,070	262,001	349,977	451,662	1,057,485
(d) Based on fixed LV ratio							
LV ratio, fixed	2,140	0.54	0.54	0.54	0.54	0.54	0.54
Land value, fixed	2,140	368,688	118,047	262,366	351,276	451,483	1,057,953

Notes: The fitted prices are derived from the baseline hedonic model, Table 3, Model 4. while fitted values for the back cast LV ratios and land values in Panel (a) are based on Model 4 in Table 5. Analogously, Panel (b) is based on Model 4 in Table A3. Panel (c) presents averages of statistics from 100 iterations in which the back casted LV ratios estimates have been randomly reshuffled. In Panel (d), the LV ratio is fixed to the average value of 0.54 for all observations.

Table 7: Fitted values for sales prices, LV ratios and land values, built 1990–1999 (extrapolated)

Variable	*N*	*mean*	*min*	*p25*	*p50*	*p75*	*max*	*Diff. 1999 sample (Table 6)*
Price	10,005	543,746	32,700	380,000	489,000	665,000	1,550,000	27.2%
Price, fitted	10,005	537,120	185,398	383,890	488,624	663,694	1,475,050	27.1%
(a) Based on back casting								
LV ratio, back casted	10,005	0.56	0.40	0.52	0.56	0.59	0.84	-3.9%
Land value, back casted	10,005	304,094	84,913	209,065	273,454	377,251	1,046,033	23.6%
(b) Based on residual method								
LV ratio, resid. method	10,005	0.58	0.33	0.51	0.58	0.63	0.96	-6.4%
Land value, resid. method	10,005	315,855	82,909	211,148	280,438	394,612	1,143,455	20.4%
(c) Based on back casting, reshuffled LV ratios								
LV ratio, back casted and reshuffled	10,005	0.54	0.50	0.53	0.54	0.55	0.58	0.0%
Land value, based on reshuffled LV	10,005	303,009	98,822	213,090	276,853	375,213	1,042,613	21.5%
(d) Based on fixed LV ratio								
LV ratio, fixed	10,005	0.54	0.54	0.54	0.54	0.54	0.54	0.0%
Land value, fixed	10,005	290,045	100,115	207,301	263,857	358,395	796,527	27.1%

Notes: This table is analogous to Table 6 except that values are calculated for the sample built 1990-1999 (extrapolation sample). The fitted values are extrapolated using coefficients estimated on the 2000–2018 sample described in Table 1, Panel (d). The last column calculates the difference in means to the 2000 –2018 sample from Table 6.

Table 8: Maricopa Market 5 CODs and Correlation Coefficients

Sample	*Correl. coeff.*	*AVM*			*Maricopa assessments*			*Diff. COD*
	AVM vs. Maricopa	*COD*	*Lower CI*	*Upper CI*	*COD*	*Lower CI*	*Upper CI*	*AVM–Maricopa*
2014–2015	0.8669	0.1338	0.1306	0.1369	0.1187	0.115	0.1226	0.0151
2014–2015, new homes	0.9265	0.1009	0.0915	0.1117	0.0762	0.0688	0.0847	0.0247
2012–2015	0.8232	0.1342	0.1312	0.1374	0.1187	0.115	0.1226	0.0155

Notes: Data are from the Maricopa 2017 assessed values and author AVM calculations. Homes aged < 16 years are defined as "new" (average age 7 yrs). CODs divide assessed values by sales in the given years at the individual property level. Assessed values labelled as "2017" take effect as of January 1, 2016, so the sales years are earlier. AVM values were calculated from hedonic regressions (Model 4 in Tables 2 and 3) and CODs are based on the avratio which equals values predicted by the regression divided by sales prices.

Table 9: FHFA Market 5 Land and Structure Values vs. AVM Values

	(1) *Prop. value, as is*	*(2)* *Prop. value, as is*	*(3)* *Maricopa assessed values, 2017*	*(4)* *Struc. value, as is*	*(5)* *Landshare prop. val. ws*	*(6)* *Landshare prop. val. ws*
AVM estimates	0.0008***		0.9119***			
	(6.48)		(5.39)			
Maricopa assessed values, 2017		0.0006***				
		(3.74)				
Depr. cost estimate				0.0012***		
				(6.02)		
LV ratio, back casted					0.2104*	
					(2.16)	
LV ratio, resid. method						0.1994*
						(2.31)
constant	-169.8571*	16.8590	-3.87e+04	-192.7145**	0.1819***	0.1851***
	(-1.99)	(0.17)	(-0.44)	(-2.72)	(4.03)	(4.74)
N	34	34	34	34	34	34
R-sq	0.840	0.559	0.726	0.715	0.269	0.304

Notes: t statistics in parentheses use robust standard errors, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. The FHFA data from Davis *et al.* (2019) are by zip code and year from 2012–2018; "as-is" measures are based on the actual characteristics of each property in the geography without any adjustment to a standard set of characteristics and "ws" is a working sample with GSE cost appraisals and passing a variety of filters. See FHFA documentation for details. We merged with median zip code-year Maricopa assessed values and with back casting results for structure price, land value ratios and property values and regressed FHFA estimates on corresponding Maricopa and back casting estimates. Merging resulted are in 34 zip code years with at least 10 transactions.

Table 10: Structure Price Regression Estimates

	(1) ln(price)	Structure price			
		(2) back casting	(3) resid. method	(4) back casted, reshuffled	(5) fixed LV ratio
ln(int. area)	0.6171***	0.1775***	0.2274***	0.1840	0.1821***
	(10.47)	(5.59)	(6.46)		(4.78)
ln(prop. age)	-0.0779***	-0.0495***	-0.0568***	-0.0301	-0.0304***
	(-7.04)	(-4.01)	(-4.22)		(-3.03)
prop. quality=4	0.0685	0.0514	0.0526	-0.0013	0.0001
	(0.96)	(1.30)	(1.33)		(0.00)
prop. quality=5	0.1550**	0.1181**	0.0922*	0.0247	0.0266
	(2.24)	(2.62)	(2.05)		(0.77)
prop. quality=6	0.1838***	0.2114***	0.1868***	0.0529	0.0541
	(2.80)	(5.59)	(4.99)		(1.48)
has pool	0.0462***	0.0274***	0.0234***	0.0149	0.0149*
	(4.53)	(3.26)	(2.81)		(2.03)
ln(lot size)	0.1917***	0.0331**	0.0271*	0.0708	0.0709***
	(8.63)	(2.17)	(1.94)		(3.66)
close to golf course	0.1039***	0.0196	-0.0020	0.0244	0.0253
	(2.82)	(0.72)	(-0.07)		(0.75)
high elevation	0.1549***	0.0209	-0.0066	0.0475	0.0470**
	(4.72)	(0.58)	(-0.17)		(2.06)
ln(dist. water)	-0.0508***	-0.0140	-0.0030	-0.0146	-0.0145
	(-2.94)	(-0.94)	(-0.20)		(-1.08)
ln(dist. CBD)	0.3859***	-0.1366	-0.0878	0.0011	0.0027
	(3.59)	(-1.05)	(-0.69)		(0.04)
nbhd. group=2	0.0327	0.0120	0.0231	0.0123	0.0123
	(1.49)	(0.40)	(0.77)		(0.75)
nbhd. group=3	0.3797***	0.0052	0.0165	0.1214	0.1208**
	(5.96)	(0.08)	(0.25)		(2.11)
year dummies	Yes	Yes	Yes	Yes	Yes
Constant	5.8570***	-0.7812	-1.4545***	-1.6225	-1.6160***
	(17.12)	(-1.69)	(-3.29)		(-7.53)
N	2,140	2,140	2,140		2,140
R-sq	0.845	0.405	0.386	0.464	0.462

Notes: t statistics in parentheses, based on robust SEs. Column (2) presents OLS regression coefficients in which the estimated structure price based on fitted LV ratios (step #5 of the algorithm) is regressed against a set of property and location-related hedonics. Column (3) is estimated using land residual assumptions. In both models, property size, age, quality, presence of a pool and the size of the lot are statistically significant. The coefficient for all other location-related attributes are not significantly different from 0. Model 4 presents average coefficients from 100 iterations in which structure prices have been derived from back casted LV ratio estimates which are randomly reshuffled within the sample in each iteration. The average coefficients for Model 4 are similar in magnitude to coefficient estimates from a model in which the LV ratios have been set to the mean value of 0.54 for all observations, rendering t-values irrelevant for Model 4.

Table 11: COD comparison

	COD	*Lower CI*	*Upper CI*	*Change COD %*
	(a) Back casting method			
Additive: prop value = land+struct value, built since 1999	0.128	0.122	0.134	
Additive: prop value = land+struct value, built since 1989	0.132	0.129	0.135	
Baseline built since 1999	0.118	0.113	0.124	
Baseline built since 1989	0.125	0.122	0.128	
	(b) Residual methhod			
Additive: prop value = land+struct value, built since 1999	0.131	0.125	0.137	-2.20%
Additive: prop value = land+struct value, built since 1989	0.133	0.130	0.136	-0.75%
	(c) LV Ratios shuffled			
Additive: prop value = land+struct value, built since 1999	0.126	0.120	0.132	1.35%
Additive: prop value = land+struct value, built since 1989	0.133	0.130	0.135	-0.38%
	(d) Fixed LV Ratio (0.54)			
Additive: prop value = land+struct value, built since 1999	0.126	0.120	na	1.35%
Additive: prop value = land+struct value, built since 1989	0.133	0.130	0.136	-0.38%

Notes: All CoD's are based on Model 4, baseline with N = 2,140 and 12,145, respectively. The main result of this tables is that for the additive model, the in-sample COD (0.128) is much higher than the baseline (0.118). The difference of 0.01 is enormous to a tax assessor and it is statistically significant in the sense that the baseline confidence interval excludes the additive model point estimate and vice versa. Land residual CODs are somewhat worse than back casting, but only by between 1 and 2%, a result that is not statistically significant. Our finding that the models do not improve assessment equity is compelling because the in-sample structures are relatively new and we estimate construction costs reliably as evaluated with FHA data. Extrapolating land values to the sample built after 1989 produces even larger CODs relative to baseline: the increase is marginally significant to assessors +0.004 (=0.132-0.128) but not statistically significant. The the fixed land value ratio model (d) produced CODs very similar to back casting and land residuals. We find similar results with random reshuffling of land value ratios: one random draw is illustrated in (c).

APPENDIX: TABLES

Table A1: Numerical solutions to back casting and land residual models

Panel A. Find an optimal structure size, S, in 2007

Parameters	*Assumed values*	*Structure size, S*	*First order condition*	*House value*	*Land Value, V*
a	1	1	5.200	136	126.0
b	1	2	4.293	152	130.7
c	0.05	3	3.625	168	134.7
k	10	4	3.109	184	138.0
d	1.1	5	2.691	200	140.9
delta (deprctn)	0.03	6	2.339	215	143.4
p	0.9	7	2.035	231	145.6
r	0.05	8	1.767	246	147.5
Variables		9	1.528	261	149.2
L	1	10	1.312	276	150.6
		11	1.115	292	151.8
		12	0.933	307	152.8
		13	0.765	322	153.7
		14	0.608	337	154.3
		15	0.461	352	154.9
		16	0.323	366	155.3
		17	0.193	381	155.5
		18	0.069	396	155.7
Optimal S*		19	-0.048	411	155.7
		20	-0.160	425	155.6
		21	-0.266	440	155.3
		22	-0.368	455	155.0

Notes: House value is the solution to equation (3); land value is the solution to equation (6) both conditional on the variables S and L and other parameters of the model. The first order condition is the derivative of equation (6) with respect to S. Optimal S* is where the FOC equals zero and it changes from positive to negative (the second order condition). The optimal structure value is 255.1 (=411-155.7).

Panel B: Model solutions for Figure 2, bundled good assumptions

Age	*Depreciated house value wrt age*	*year: new constrction in 2007*	*Actual HPI, Mkt #5, 2007=1.0*	*Depreciated prop. value over time*	*land value*	*land value ratio: bundled assumption*
0	411	2007	1.44	411	156	0.379
1	402	2008	1.22	341	132	0.387
2	394	2009	0.99	271	107	0.395
3	387	2010	0.93	249	100	0.403
4	379	2011	0.89	235	96	0.411
5	372	2012	1.00	257	108	0.419
6	364	2013	1.16	293	125	0.427
7	357	2014	1.21	300	131	0.435
8	351	2015	1.26	306	136	0.444
9	344	2016	1.30	310	140	0.452
10	338	2017	1.37	321	148	0.461
11	332	2018	1.45	333	156	0.470

Notes: For a property with an HBU new structure in 2007, the land value ratios are calculated with bundled good assumptions. The first two columns give the depreciated value of the property using 3% per year depreciation. The actual HPI is property value over time calculated from baseline AVM year coefficients for market 5 (Table 3, Model 4). Depreciated property value over time is column 2 updated with the HPI. E.g., 341 = 402*1.22/1.44). Land value is property value minus depreciated structure value.

Panel C: Model solutions for Figure 2, land residual assumptions

year: new constrction in 2007	*construct. cost index*	*optimal structure cost new over time*	*Deprec'td structure cost*	*Depreciated prop. value over time*	*land value ratio*
2007	1.00	255	255	411	0.379
2008	1.01	258	250	341	0.267
2009	1.02	260	245	271	0.096
2010	1.03	263	240	249	0.035
2011	1.04	265	235	235	-0.004
2012	1.05	268	231	257	0.104
2013	1.06	271	226	293	0.228
2014	1.07	273	222	300	0.261
2015	1.08	276	217	306	0.290
2016	1.09	279	213	310	0.314
2017	1.10	282	209	321	0.350
2018	1.12	285	205	333	0.385

Notes: For a property with an HBU new structure in 2007, the land value ratios are calculated with land residual assumptions. Structure value in any year is the depreciated cost to rebuild in that year: building cost appreciation is given in column 2 and depreciation is 3%/year. Property value is the same as Panel B.

Table A2: Price and LV ratios, back casted and land residual, per year

Year or nbhd.	*Variable*	*N*	*mean*	*SD*	*p25*	*p50*	*p75*
2007	price	169	865,744	284,523	631,500	825,000	1,000,000
	LV ratio, back casted	169	.53	.09	.46	.54	.60
	LV ratio, at sale	169	.64	.05	.59	.63	.67
2008	price	126	756,472	296,729	525,000	707,500	875,000
	LV ratio, back casted	126	.54	.08	.48	.55	.60
	LV ratio, at sale	126	.60	.07	.55	.60	.65
2009	price	146	593,253	219,200	425,000	542,500	705,000
	LV ratio, back casted	146	.52	.08	.46	.52	.59
	LV ratio, at sale	146	.50	.08	.42	.52	.56
2010	price	151	547,345	206,996	390,000	530,000	645,000
	LV ratio, back casted	151	.53	.07	.49	.52	.58
	LV ratio, at sale	151	.46	.08	.39	.47	.52
2011	price	182	573,009	235,574	395,000	550,000	715,000
	LV ratio, back casted	182	.54	.08	.49	.53	.59
	LV ratio, at sale	182	.44	.09	.38	.46	.50
2012	price	183	634,861	211,463	500,000	585,000	724,332
	LV ratio, back casted	183	.54	.10	.49	.53	.59
	LV ratio, at sale	183	.49	.09	.42	.50	.54
2013	price	157	706,008	274,614	485,000	640,000	855,963
	LV ratio, back casted	157	.55	.08	.51	.54	.60
	LV ratio, at sale	157	.54	.08	.49	.56	.59
2014	price	205	630,182	297,034	393,190	555,000	765,000
	LV ratio, back casted	205	.50	.14	.33	.53	.60
	LV ratio, at sale	205	.48	.13	.33	.52	.59
2015	price	198	674,299	296,873	419,990	604,000	842,500
	LV ratio, back casted	198	.52	.11	.42	.55	.61
	LV ratio, at sale	198	.54	.12	.45	.57	.62
2016	price	200	763,602	299,584	549,228	687,500	942,500
	LV ratio, back casted	200	.57	.08	.53	.58	.62
	LV ratio, at sale	200	.60	.07	.55	.61	.65
2017	price	209	746,236	289,804	558,000	682,000	875,000
	LV ratio, back casted	209	.57	.09	.52	.57	.63
	LV ratio, at sale	209	.59	.09	.54	.61	.65
2018	price	214	779,808	293,957	549,000	726,150	995,000
	LV ratio, back casted	214	.56	.10	.52	.58	.63

LV ratio, at sale	214	.59	.10	.53	.62	.66

Notes: Statistics are for land value ratios calculated from algorithm steps #1 and #2, before modelling the ratios.

Table A3: Regression Estimates LV ratio based on residual method

	(1) LV ratio, resid.	(2) LV ratio, resid.	(3) LV ratio, resid.	(4) LV ratio, resid.	(5) LV ratio, resid.
ln(int. area)	-0.100***	-0.123***	-0.073***	-0.096***	
	(-9.99)	(-14.07)	(-12.11)	(-26.78)	
ln(prop. age)	0.034***	0.030***	0.045***	0.041***	
	(14.45)	(14.38)	(30.07)	(45.60)	
prop. quality=4	-0.135***	-0.093***	-0.144***	-0.103***	-0.110***
	(-8.92)	(-6.99)	(-15.67)	(-18.89)	(-24.00)
prop. quality=5	-0.174***	-0.121***	-0.177***	-0.125***	-0.140***
	(-11.19)	(-8.80)	(-18.77)	(-22.17)	(-29.44)
prop. quality=6	-0.251***	-0.183***	-0.263***	-0.198***	-0.214***
	(-14.78)	(-12.18)	(-25.38)	(-31.99)	(-40.65)
has pool	-0.009**	-0.008**	-0.017***	-0.017***	-0.017***
	(-1.99)	(-2.24)	(-6.57)	(-11.18)	(-13.13)
ln(lot size)	0.088***	0.077***	0.078***	0.067***	
	(22.05)	(22.02)	(32.09)	(46.74)	
close to golf course	0.088***	0.052***	0.081***	0.046***	0.035***
	(9.30)	(6.26)	(14.27)	(13.53)	(12.10)
high elevation	0.069***	0.061***	0.074***	0.066***	0.063***
	(15.36)	(14.72)	(27.07)	(39.29)	(44.60)
ln(dist. water)	-0.040***	-0.018***	-0.045***	-0.024***	-0.023***
	(-11.72)	(-5.81)	(-21.96)	(-19.10)	(-21.18)
ln(dist. CBD)	0.046***	0.195***	0.036***	0.194***	0.202***
	(2.76)	(11.63)	(3.57)	(27.80)	(33.44)
nbhd. group=2		0.016***		0.011***	0.016***
		(3.47)		(5.82)	(9.82)
nbhd. group=3		0.158***		0.154***	0.166***
		(25.67)		(60.92)	(77.18)
Year dummies	No	No	Yes	Yes	Yes
int. floor area (1,000 sqft.)					-0.032***
					(-5.66)
int. floor area, squared					0.001
					(0.81)
prop. age, 100 years					0.522***
					(12.17)
prop. age, squared					2.483***
					(9.30)
lot size (10,000 sqft.)					0.078***
					(30.39)
lotsize, squared					-0.008***
					(-18.07)
Constant	0.761***	0.287***	0.835***	0.340***	0.207***
	(8.59)	(3.53)	(15.48)	(10.14)	(9.56)
Observations	2140	2140	2140	2140	2140
R^2	0.473	0.603	0.811	0.934	0.954

t statistics in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table A4: Structure Price Regression estimates, built since 1989 (extrapolated)

	(1) ln(price)	*Structure price*			
		(2) back casting	*(3) resid. method*	*(4) back casted, reshuffled*	*(5) fixed LV ratio*
ln(int. area)	0.5904***	0.1126***	0.1404***	0.1444	0.1429***
	(15.48)	(5.39)	(6.78)		(5.38)
ln(prop. age)	-0.1041***	-0.0852***	-0.1038***	-0.0361	-0.0363***
	(-5.73)	(-4.51)	(-4.89)		(-2.89)
prop. quality=4	0.0630*	0.0514***	0.0490***	-0.0034	-0.0025
	(1.78)	(4.04)	(3.46)		(-0.16)
prop. quality=5	0.2188***	0.1168***	0.0859***	0.0378	0.0391*
	(5.61)	(5.78)	(4.09)		(1.93)
prop. quality=6	0.3086***	0.2261***	0.1959***	0.0859	0.0865***
	(6.69)	(8.70)	(7.36)		(3.21)
has pool	0.0346***	0.0175***	0.0136***	0.0053	0.0053
	(4.47)	(4.42)	(3.41)		(1.01)
ln(lot size)	0.1762***	0.0113	0.0045	0.0596	0.0597***
	(9.69)	(0.60)	(0.24)		(4.41)
close to golf course	0.1198***	-0.0032	-0.0281	0.0205	0.0212
	(3.55)	(-0.13)	(-1.12)		(0.80)
high elevation	0.0921***	0.0268	0.0060	0.0271	0.0267*
	(3.85)	(1.28)	(0.27)		(1.84)
ln(dist. water)	-0.0284*	-0.0043	0.0061	-0.0119	-0.0118
	(-1.84)	(-0.47)	(0.70)		(-1.14)
ln(dist. CBD)	0.1993*	-0.1267	-0.0903	-0.0013	-0.0000
	(1.83)	(-1.07)	(-0.76)		(-0.00)
nbhd. group=2	0.1034***	-0.0031	0.0043	0.0268	0.0268**
	(4.95)	(-0.18)	(0.25)		(2.54)
nbhd. group=3	0.2966***	-0.0282	-0.0296	0.0892	0.0886**
	(5.76)	(-0.72)	(-0.73)		(2.25)
year dummies	Yes	Yes	Yes	Yes	Yes
Constant	6.6899***	-0.1145	-0.5267	-1.2069	-1.2003***
	(15.26)	(-0.22)	(-1.01)		(-4.19)
N	12,145	12,145	12,145	12,145	12,145
R-sq	0.837	0.403	0.390	0.4705	0.469

Notes: t statistics in parentheses, based on robust SE.

Table A5: CoD comparison

	Back casting method			Residual methhod			Change COD
	COD	*Lower CI*	*Upper CI*	*COD*	*Lower CI*	*Upper CI*	*%*
			Property Age > 10 years				
Additive: prop value = land+struct value, 1999	0.103	0.098	0.11	0.105	0.099	0.111	-1.30%
Baseline built since 1999	0.097	0.092	0.104	0.097	0.092	0.104	0.00%
			Year of sale > 2011				
Additive: prop value = land+struct value, 1999	0.126	0.12	0.133	0.128	0	0.135	-1.50%
Baseline built since 1999	0.113	0.107	0.119	0.113	0.107	0.119	0.00%
			Exclude the most expensive neighborhoods (nbhd. group 3)				
Additive: prop value = land+struct value, 1999	0.119	0.114	0.124	0.121	0	0	-1.50%
Baseline built since 1999	0.108	0.103	0.113	0.108	0.103	0.113	0.00%
			Exclude the least expensive neighborhoods (nbhd. group 1)				
Additive: prop value = land+struct value, 1999	0.124	0.118	0.131	0.126	0	0	-1.00%
Baseline built since 1999	0.117	0.111	0.124	0.117	0.111	0.124	0.00%

Notes: This table shows that several subsamples produce only small differences in the 0.007 to 0.012 range, establishing the robustness of results in Table 11.